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Contents

Estimation of conditional distribution function under dependent random censored data

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The aim of paper is considering the problem of estimation of conditional survival function in the case of right random censoring with presence of covariate.

Let's consider the case when the support of covariate C is the interval $[0,1]$ and we describe our results on fixed design points $0 \leq x_1 \leq x_2 \leq ... \leq x_n \leq 1$ at which we consider responses (survival or failure times) $X_1, ..., X_n$ and censoring times $Y_1, ..., Y_n$ of identical objects, which are under study. These responses are independent and nonnegative random variables (r.v.-s) with conditional distribution function (d.f.) at x_i, $F_{x_i}(t) = P(X_i \leq t / C_i = x_i)$. They are subjected to random right censoring, that is for X_i there is a censoring variable Y_i with conditional d.f. $G_{x_i}(t) = P(Y_i \leq t / C_i = x_i)$ and at n-th stage of experiment the observed data is $S^{(n)} = \{(Z_i, \delta_i, C_i),\ 1 \leq i \leq n\}$, where $Z_i = \min(X_i, Y_i)$, $\delta_i = I(X_i \leq Y_i)$ with $I(A)$ denoting the indicator of event A.

Note that in sample $S^{(n)}$ r.v. X_i is observed only when $\delta_i = 1$. Commonly, in survival analysis to assume independence between the r.v.-s X_i and Y_i conditional on the covariate C_i. But, in some practical situations, this assumption does not hold. Therefore, in this article we consider a dependence model in which dependence structure is described through copula function. So let $S_x(t_1, t_2) = P(X_x > t_1, Y_x > t_2)$, t_1, $t_2 \geq 0$, the joint survival function of the response X_x and the censoring variable Y_x at x. Then the marginal survival functions are $S_x^X(t) = 1 - F_x(t) = S_x(t, 0)$ and $S_x^Y(t) = 1 - G_x(t) = S_x(0, t)$, $t \leq 0$. We suppose that the marginal d.f.-s F_x and G_x are continuous. Then according to the Theorem of Sclar (see, [1]), the joint survival function $S_x(t_1, t_2)$ can be expressed as

$$S_x(t_1, t_2) = C_x(S_x^X(t_1), S_x^X(t_2)), \quad t_1, t_2 \geq 0, \tag{1}$$

where $C_x(u, v)$ is a known copula function depending on x, S_x^X and S_x^Y in a general way. We consider estimator of d.f. F_x which is equivalent

to the relative-risk power estimator [2, 3] under independent censoring case.

Assume that at the fixed design value $x \in (0,1)$, C_x in (1) is Archimedean copula, i.e.

$$S_x(t_1,t_2) = \varphi_x^{[-1]}\left(\varphi_x(S_x^X(t_1)) + \varphi_x\left(S_x^Y(t_2)\right)\right), \quad t_1,t_2 \geq 0, \tag{2}$$

where, for each x, $\varphi_x : [0,1] \to [0,+\infty]$ is a known continuous, convex, strictly decreasing function with $\varphi_x = 0$. We assume that copula generator function φ_x is strict, i.e. $\varphi_x(0) = \infty$ and φ_x^{-1} is a inverse of φ_x. From (2), it follows that

$$\begin{aligned} P(Z_x > t) = 1 - H_x(t) = \overline{H_x(t)} = S_x^Z(t) = S_x(t,t) \\ = \varphi_x^{-1}(\varphi_x(S_x^X(t)) + \varphi_x(S_x^Y(t))), \quad t \geq 0, \end{aligned} \tag{3}$$

Let $H_x^{(1)}(t) = P(Z_x \leq t, \delta_x = 1)$ be a subdistribution function and $\Lambda_x(t)$ is crude hazard function of r.v. X_x subjecting to censoring by Y_x,

$$\Lambda_x(dt) = \frac{P(X_x \in dt, X_x \leq Y_x)}{P(X_x \geq t, Y_x \geq t)} = \frac{H_x^{(1)}(dt)}{S_x^Z(t-)}. \tag{4}$$

From (4) one can obtain following expression of survival function S_x^X:

$$S_x^X(t) = \varphi_x^{-1}\Big[-\int_0^t \varphi_x'\big(S_x^Z(u)\big)\, dH_x^{(1)}(u)\Big], \quad t \geq 0. \tag{5}$$

In order to constructing the estimator of S_x^X according to representation (5), we introduce smoothed estimators of S_x^Z, $H_x^{(1)}$ and regularity conditions for them. We use the Gasser–Müller weights

$$w_{ni}(x,h_n) = \frac{1}{q_n(x,h_n)} \int_{x_{i-1}}^{x_i} \frac{1}{h_n}\,\pi\left(\frac{x-z}{h_n}\right) dz, \quad i = 1,...,n, \tag{6}$$

with

$$q_n(x,h_n) = \int_0^{x_n} \frac{1}{h_n}\,\pi\left(\frac{x-z}{h_n}\right) dz,$$

where $x_0 = 0$, π is a known probability density function (kernel) and $\{h_n, n \geq 1\}$ is a sequence of positive constants, tending to zero as $n \to \infty$, called bandwidth sequence.

Let's introduce the weighted estimators of H_x, S_x^Z and $H_x^{(1)}$ respectively as

$$H_{xh}(t) = \sum_{i=1}^{n} w_{ni}(x, h_n)\, I(Z_i \le t), \quad S_{xh}^Z(t) = 1 - H_{xh}(t),$$
$$H_{xh}^{(1)}(t) = \sum_{i=1}^{n} w_{ni}(x, h_n)\, I(Z_i \le t, \delta_i = 1). \tag{7}$$

Then plugging estimators (6) and (7) in (5), we obtain the following intermediate estimator of S_x^X:

$$S_{xh}^X(t) = 1 - F_{xh}(t) = \varphi_x^{-1}\Big[- \int_0^t \varphi_x'\big(S_x^Z(u)\big)\, dH_x^{(1)}(u)\Big], \quad t \ge 0.$$

In this work we propose the next extended analogue of estimator introduced in [2, 3]:

$$\widehat{S}_{xh}^X(t) = \varphi_x^{-1}\big[\varphi\big(S_{xh}^Z(t)\big) \cdot \mu_{xh}(t)\big] = 1 - \widehat{F}_{xh}(t), \tag{8}$$

where

$$\mu_{xh}(t) = \varphi\big(S_{xh}^X(t)\big)/\varphi\big(\widetilde{S}_{xh}^Z(t)\big),$$
$$\varphi\big(S_{xh}^X(t)\big) = - \int_0^t \varphi_x'\big(S_{xh}^Z(u)\big)\, dH_{xh}^{(1)}(u),$$
$$\varphi\big(\widetilde{S}_{xh}^Z(t)\big) = - \int_0^t \varphi_x'\big(S_{xh}^Z(u)\big)\, dH_{xh}(u).$$

In order to investigate the estimate (6) we introduce some conditions. For the design points x_1, ..., x_n, denote

$$\underline{\Delta_n} = \min_{1\le i\le n} (x_i - x_{i-1}), \quad \overline{\Delta_n} = \max_{1\le i\le n} (x_i - x_{i-1}).$$

For the kernel π, let

$$\|\pi\|_2^2 = \int_{-\infty}^{\infty} \pi^2(u)\, du, \quad m_\nu(\pi) = \int_{-\infty}^{\infty} u^\nu\, \pi(u)\, du, \quad \nu = 1, 2.$$

Moreover, we use next assumptions on the design and on the kernel function:

(A1) As $n \to \infty$, $x_n \to 1$, $\underline{\Delta_n} = O(\frac{1}{n})$, $\overline{\Delta_n} - \underline{\Delta_n} = o(\frac{1}{n})$.

(A2) π is a probability density function with compact support $[-M, M]$ for some $M > 0$, with $m_1(\pi) = 0$ and $|\pi(u) - \pi(u')| \leq C(\pi)\,|u - u'|$, where $C(\pi)$ is some constant.

Let $T_{H_x} = \inf\{t \geq 0 : H_x(t) = 1\}$. Then $T_{H_x} = \min(T_{F_x}, T_{G_x})$. For our results we need some smoothnees conditions on functions $H_x(t)$ and $H_x^{(1)}(t)$. We formulate them for a general (sub)distribution function $N_x(t)$, $0 \leq x \leq 1$, $t \in R$ and for a fixed $T > 0$.

(A3) $\frac{\partial^2}{\partial x^2} N_x(t) = \ddot{N}_x(t)$ exists and is continuous in $(x, t) \in [0, 1] \times [0, T]$.

(A4) $\frac{\partial^2}{\partial t^2} N_x(t) = N_x''(t)$ exists and is continuous in $(x, t) \in [0, 1] \times [0, T]$.

(A5) $\frac{\partial^2}{\partial x\,\partial t} N_x(t) = \dot{N}_x'(t)$ exists and is continuous in $(x, t) \in [0, 1] \times [0, T]$.

(A6) $\frac{\partial \varphi_x(u)}{\partial u} = \varphi_x'(u)$ and $\frac{\partial^2 \varphi_x(u)}{\partial u^2} = \varphi_x''(u)$ are Lipschitz in the x-direction with a bounded Lipschitz constant and $\frac{\partial^3 \varphi_x(u)}{\partial u^3} = \varphi_x'''(u)$ exists and is continuous in $(x, u) \in [0, 1] \times (0, 1]$.

Under conditions (A1)-(A6) we derive an almost sure representation result of the difference $\widehat{F}_{xh}(t) - F_x(t)$ with rate and weak convergence results for the process $\left\{(nh_n)^{1/2}\left[\widehat{F}_{xh}(\cdot) - F_x(\cdot)\right],\ 0 \leq t \leq T\right\}$ to the Gaussian processes.

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Stability analysis of retrial queueing systems based on the synchronization method

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We consider two retrial queueing models M_1 and M_2 in which primary customers arrive according to a regenerative flow $\{X(t), t \geq 0\}$ of

rate λ_X (Afanaseva and Bashtova [2]). Let $\{\theta_j\}_{j=1}^{\infty}$ be a sequence of regeneration points for $X(t)$, $\tau_j = \theta_{j+1} - \theta_j$ is the j-th regeneration period and $\xi_j = X(\theta_{j+1}) - X(\theta_j)$ ($j = 0, 1, ..., \theta_0 = 0$). Assume $\mathbf{E}\,\tau_1 < \infty$, $\mathbf{E}\,\xi_1 < \infty$ then w.p. 1

$$\lambda_X = \lim_{t\to\infty} \frac{X(t)}{t} = \frac{\mathbf{E}\,\xi_1}{\mathbf{E}\,\tau_1}.$$

There are m identical servers in the systems and service times form a sequence $\{\eta_n\}_{n=1}^{\infty}$ of independent identically distributed (iid) random variables with c.d.f. $B(x)$ and finite mean $b = \int_0^{\infty} x\,dB(x)$. An arriving customer finding one or more servers idle obtains service immediately. Customers who find all servers busy go directly to the orbit and start generating requests for service. For the model M_1 we assume that the flow of requests for service from the orbit is a doubly stochastic Poisson process (DSPP) (see Grandell [3]) with a random intensity $\nu(Z(t))$. Here $Z(t)$ is the number of customers on the orbit at time t. If there is an idle server at time of the request from the orbit then the service one from $Z(t)$ customers begins.

In the model M_2 the repeated requests are realized through iid random intervals $\{\zeta_n\}_{n=0}^{\infty}$ with $\mathbf{E}\,\zeta_n = \nu^{-1}$ independently of the number of customers on the orbit. Thus, the rate of the flow of the repeated requests is a constant that is equal to ν. For the both models M_1 and M_2 we consider a stochastic process $q(t)$ that is the number of customers in the system at time t. We will call this process a stable one if there exists the limit

$$\lim_{t\to\infty} \mathbf{P}(q(t) \le x) = \Phi(x),$$

where $\Phi(x)$ is a d.f. not depending on any initial state of the system.

Condition 1.

$$\mathbf{P}(\xi_1 = 0, \tau_1 > 0) + \mathbf{P}(\xi_1 = 1, \tau_1 - t_1 > \eta_1) > 0,$$

where $\theta_1 + t_1$ – is the arrival time and η_1 the service time of the unique customer on the regeneration period (θ_1, θ_2).

Note, this condition provides the hit of the process $q(t)$ to zero state from any initial state of the system with positive probability.

Condition 2. For the model M_2 the random variable ζ_n has the second exponential phase.

This means that

$$\zeta_n = \zeta_n^{(1)} + \zeta_n^{(2)},$$

where $\zeta_n^{(1)}$ and $\zeta_n^{(2)}$ are independent random variables and

$$\mathbf{P}(\zeta_n^{(2)} > x) = e^{-\gamma x}, \quad \gamma > 0.$$

Under Condition 1 the process $q(t)$ is a regenerative one for the model M_1 and for the model M_2 it is valid under additional Condition 2.

Stability Theorem for the Model M_2

Let $N(t)$ be a counting process for the sequence $\{\zeta_n\}_{n=0}^{\infty}$, i.e.

$$N(t) = \max\Big\{k \geq 0 : \sum_{j=1}^{k} \zeta_j \leq t\Big\}.$$

Consider m-server system with refusals and a regenerative input flow $U(t) = X(t) + N(t)$, i.e. $Reg|G|m|0$. Let $n(t)$ be the number of busy servers at time t in this system and

$$\lim_{k\to\infty} \mathbf{P}(n(t_k) = j) = p_j, \quad j = 0, 1, \ldots,$$

where $\{t_k\}_{k=1}^{\infty}$ is the sequence of moments of jumps of the input flow $U(t)$. We define the traffic rate for the model M_2 as follows:

$$\rho_2 = \frac{\lambda_X}{(\lambda_X + \nu)(1 - p_m)}.$$

Theorem 1. Let Conditions 1 and 2 be fulfilled. The process $q(t)$ is a stable one iff $\rho_2 < 1$.

The proof is based on synchronization of $X(t)$ and auxiliary process $\widetilde{Y}(t)$ that is the number of served customers up to time t in the auxiliary system $\widetilde{M_2}$ in which always there are customers on the orbit.

Corollary 1. Let $X(t)$ and $N(t)$ be Poisson processes with rate λ and ν respectively. Then $q(t)$ is a stable process iff

$$\frac{\lambda}{\lambda + \nu} < \frac{\sum\limits_{j=0}^{m-1} \frac{\alpha^j}{j!}}{\sum\limits_{j=0}^{m} \frac{\alpha^j}{j!}}, \tag{1}$$

where $\alpha = b\,(\lambda + \nu)$.

Stability Theorem for the Model M_1

Theorem 2. Let Condition 1 be fulfilled. The intensity of repeated requests is nondecreasing function $\nu(j)$ and $\lim\limits_{j\to\infty} \nu(j) = \infty$. Then $q(t)$ is a stable process iff

$$\rho = \frac{\lambda b}{m} < 1.$$

Now consider the case

$$\lim_{j\to\infty} \nu(j) = \nu < \infty. \tag{2}$$

Corollary 2. Let $X(t)$ be a Poisson process, $\nu(j)$ is non-decreasing function and (2) holds. Then for the model M_1 the necessary and sufficient condition is of the form (1).

Conclusion. We considered the generalization of the classical retrial systems. The pioneering studies of retrial queues present the concept of "retrial time" as an alternative to the models of telephone systems queues with refusals (see [2] and literature there). It was assumed for retrial models that each customer on the orbit generates a flow of repeated request independently of the rest customers in the retrial group. Thus in the classical retrial policy we have for the model M_1 the intensity $\nu(j) = \nu_j$. The second class contains models with constant retrial rate. This constant retrial policy was introduced by Fayolle [5]. Since Fayolle, there has been a rapid growth in the literature (see e.g. [4], [6]). Our model M_1 belongs to this class but we assume that input flow is a regenerative one and intervals between repeated requests from the orbit have an arbitrary distribution.

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Optimal portfolio construction with two-sided weight constraints and commission

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We consider the problem of finding the optimal portfolio with two-sided constraints for weights and with commission under certainty. For the Markowitz model (portfolios without short positions) a complete efficient algorithmic solution of the problem is proposed. A heuristic efficient algorithm for solving this problem for the Black's model (portfolios with short positions) is also proposed. Under uncertainty the financial analyst usually considers the most likely scenarios of the possible completion of the transaction. In that case, the analysis of individual scenarios is carried out under certainty. The condition of certainty means that the investor knows both the current and future prices of assets (based on price forecasts) and income. The investigation of portfolio transactions under certainty is definitely useful and widely used by accountants and auditors in the analysis of closed transactions. It is significant that unlike the ideal case without commission, the task of choosing the optimal portfolio for Black's model is nonsmooth. Portfolio analysis with commission under uncertainty was investigated in detail in [1, 2].

In what follows, we consider one-period portfolio transactions with a fixed and finite investment horizon. In addition, we consider only the investment portfolios i.e. portfolios for which the proceeds from the short sales do not cover the costs of opening the long positions of the portfolio. For simplicity, assume that the dividends will not be paid separately. Note that under certainty the rational investor chooses portfolio with the highest return.

Suppose that we have n assets $A_1, \ldots, A_n$. Let r_k denote the price return of A_k. The portfolio will be denoted by the vector of asset weights $\boldsymbol{x} = (x_1, \ldots, x_n)$: $\sum_{k=1}^{n} x_k = 1$ (the budget constraint). If there are no commission costs, it is well known that the portfolio return $r(\boldsymbol{x})$ is the weighted average of the individual asset returns $r(\boldsymbol{x}) = \sum_{k=1}^{n} x_k\, r_k$.

The investor can not quite arbitrarily choose these weights for two reasons. First, the portfolio weights must satisfy the budget constraint. Secondly, in many markets there are strict limitations on the size of short positions. For example, institutional investors such as insurance companies, pension funds and some credit institutions are obliged to adhere to legislation requirements for assets weights of different classes (such as government securities, shares of companies, real estate etc.).

Specifying the class of admissible portfolios defines a particular optimization problem. Usually this class is defined by a system of equations and inequalities.

First we consider the problem of selecting the optimal portfolio with two-sided constraints and no commission. This problem is not trivial, but not difficult to solve in practical terms (see Remark 3 below). This problem is formulated as follows.

Problem 1. For given returns $r_1, r_2, \ldots, r_n$ and $\boldsymbol{a} = (a_1, \ldots, a_n)$, $\boldsymbol{b} = (b_1, \ldots, b_n)$

$$\text{maximize} \quad r(\boldsymbol{x}) = r_1\, x_1 + r_2\, x_2 + \ldots + r_n\, x_n \tag{1}$$

$$\text{subject to} \qquad x_1 + x_2 + \ldots + x_n = 1 \quad \text{and} \tag{2}$$

$$a_1 \le x_1 \le b_1, \quad a_2 \le x_2 \le b_2, \ldots, a_n \le x_n \le b_n. \tag{3}$$

The following theorem contains the necessary and sufficient condition for the solution existence of problem 1. Let $A = a_1 + a_2 + \ldots + a_n$ and $B = b_1 + b_2 + \ldots + b_n$.

Theorem 1. *The problem 1 has a solution if and only if $A \le 1 \le B$. Moreover, when $A \ne B$ an admissible portfolio can be found by the formula*

$$\boldsymbol{x} = \boldsymbol{a} + \left(\frac{1 - A}{B - A}\right) (\boldsymbol{b} - \boldsymbol{a}).$$

If the portfolio transaction is opened with a commission α and closed with a commission β, then according to [1], [2] the portfolio return has

the form

$$r_{\alpha,\beta}(\boldsymbol{x}) = \frac{1}{1+\alpha\sum\limits_{k=1}^{n}|x_k|}\left(\sum_{k=1}^{n} x_k\, r_k - \sum_{k=1}^{n}(\alpha+\beta+\beta\, r_k)\,|x_k|\right). \qquad (4)$$

Now consider the case when the investor can open only long positions (the Markowitz model), in other words, the investor forms portfolios with non-negative weights. In that case, the optimization problem 1 is formulated as follows

Problem 2. For given returns r_1, r_2, ..., r_n and $a_i \geq 0$, $b_i \leq 1$, $i = 1, 2, \ldots, n$

$$\text{maximize} \quad r_{\alpha,\beta}(\boldsymbol{x}) = a_{\alpha,\beta}\, r(\boldsymbol{x}) - b_{\alpha,\beta}$$

subject to (2) and (3), where $a_{\alpha,\beta} = \frac{1-\beta}{1+\alpha}$, $b_{\alpha,\beta} = \frac{\alpha+\beta}{1+\alpha}$.

Recall that the linear function (note that the portfolio return is a linear function of the portfolio weights) has the largest value on the boundary of the function domain. Since there is one equality constraint and $2n$ inequality constraints, then, at least, $n-1$ components of the optimal portfolio must satisfy the boundary conditions. The solution of Problem 2 (under the conditions of Theorem 1) may be found efficiently by the following general algorithm (the Swap Algorithm). Suppose that for an admissible portfolio $\boldsymbol{x}$ there exists a pair of components x_i and x_j, that do not satisfy the boundary equalities, let for example $x_i < b_i$ and $a_j < x_j$. Assume also that $r_j < r_i$. Then the swap (or exchange) of the assets A_i and A_j is possible. This swap allows to increase the portfolio return. The main idea of the swap is to sell an amount (not necessary integer) of the asset A_j (i.e., we decrease the weight x_j by a certain amount $h > 0$) and to buy A_i on the amount of revenue from the sale of A_j (i.e., we increase the weight x_i by the same amount $h > 0$). It is easy to see that the swap preserves the budget constraint. Note that the swap will preserve the boundary conditions, if h satisfies the inequalities $a_j \leq x_j - h$ and $x_i + h \leq b_i$ or, equivalently $h \leq x_j - a_j$ and $h \leq b_i - x_i$. The swap increases the portfolio return by the value $\Delta r_{ij} = h\, a_{\alpha,\beta}\,(r_i - r_j) > 0$. At the same time the extremal swap with $h = \min\{x_j - a_j, b_i - x_i\}$ gives the greatest growth of the portfolio return.

Remark 1. If all assets have the same return: $r_1 = r_2 = \cdots = r_n = r_0$ then for the Markowitz model all portfolios have the same return $r_{\alpha,\beta}(\boldsymbol{x}) = a_{\alpha,\beta}\, r_0 - b_{\alpha,\beta}$. In that case, the investor is indifferent to

the choice of a particular portfolio, provided that the received return is positive and satisfactory for the investor.

Remark 2. Let $r_i = \max\limits_{k=1,\dots,n} \{r_k\}$ and $b_i = 1$. Then the Problem 2 admits a trivial solution. Namely, the investor invests all the money in the asset A_i with the highest return, provided that the received return is positive and satisfactory for the investor.

Let us consider the case when the investor can open short positions (the Black's model). Then the optimization problem 1 is formulated as follows

Problem 3. For given returns $r_1, r_2, \dots, r_n$ and $\boldsymbol{a} = (a_1, \dots, a_n)$, $\boldsymbol{b} = (b_1, \dots, b_n)$ maximize $r_{\alpha,\beta}$ (see (4)) subject to (2) and (3).

Under the conditions of Theorem 1, the Problem 3 always has a solution, since one seeks the maximum of a continuous function on a compact set. Note that the return of the optimal portfolio should be positive and satisfactory for the investor.

To solve this problem, we propose the following heuristic algorithm. Renumber the assets so that their returns are located in nonincreasing order: $r_1 \geq r_2 \geq \ldots \geq r_n$. Apparently, in a typical situation the optimal portfolio $\boldsymbol{x}^* = (x_1^*, x_2^*, \dots, x_n^*)$ has the following property: there is a k such that $x_1^*, x_2^*, \dots, x_k^* \geq 0$ and $x_{k+1}^*, \dots, x_n^* \leq 0$. This allows to reduce the solution of the Problem 3 to the solution of n smooth problems. Namely, for each $k = 1, 2, \dots, n$ we solve the Problem 3 under the conditions that $x_1, x_2, \dots, x_k \geq 0$ and $x_{k+1}, x_{k+2}, \dots, x_n \leq 0$ and then we choose the solution with the highest return from the resulting n solutions.

Remark 3. If $\alpha = \beta = 0$ then the Swap Algorithm is applicable to Black's model. Moreover, for the Markowitz model, the solution of Problem 2 coincides with the solution of a similar problem without commission.

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Multi-period Markowitz model and optimal self-financing strategy with commission

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The goal of any financial transaction in the securities market is to achieve maximum income and to increase the initial capital. Optimal portfolio selection problem is one of the basic research fields in modern financial economics especially in the theory of portfolio analysis. In multi-period portfolio transactions with transaction costs the problem of portfolio selection becomes nontrivial and more hard to solve. In this work for the Markowitz model (only long positions are allowed for this model, see [1, 2]) we solve the problem of choosing the optimal multi-period self-financing portfolio strategy with commission under certainty (we give a complete proof for this solution). In other words, we are looking for a strategy for which all the released money from the sale of some assets will be invested in the acquisition of other assets in order to maximize the portfolio value at the end of the investment horizon.

We emphasize that the solution of this problem is nontrivial and may differ from its solution without commission. One-period portfolio analysis with commission under uncertainty was investigated in detail in [3, 4].

Let the market consist of n assets $A_1, A_2, \ldots, A_n$. The portfolio at time t will be denoted by the position vector $\boldsymbol{z}(t) = (z_1(t), z_2(t), \ldots, z_n(t))$, where $z_k(t) \geq 0$ is the position of A_k (note that $z_k(t)$ is the amount of A_k in the portfolio at the time t). Let $p_k(t)$ be the asset price of A_k at time t. Then the market state at any time t is specified by the n-dimensional price vector $\boldsymbol{p}(t) = (p_1(t), p_2(t), \ldots, p_n(t))$.

In what follows, we consider multi-period portfolio transactions with fixed and finite investment horizon. We assume that changes in asset prices occur only at discrete instants of time.

By strategy we mean a sequential restructuring of the portfolio (formed at the time $t_0 = 0$ with an initial capital) at the moments $t = t_0, t_1, \ldots, t_N$, in order to maximize the portfolio value at the time t_N. We denote the strategy by

$$\boldsymbol{Z}_{[t_0,t_N]} = \{\boldsymbol{z}(t_0), \boldsymbol{z}(t_1), \ldots, \boldsymbol{z}(t_{N-1})\}. \tag{1}$$

The strategy $\boldsymbol{Z}_{[t_0,t_N]}$ is called admissible if all the released money from the sale of some assets is invested in the acquisition of other assets.

The optimization problem with commission α is formulated as follows:

For the investment horizon $[t_0, t_N]$ *and the price forecast* $[p_k(t_i)]_{k=1,n}^{i=0,N}$ *find a strategy*

$$\boldsymbol{Z}^*_{[t_0,t_N]} = \{\boldsymbol{z}^*(t_0), \boldsymbol{z}^*(t_1), \ldots, \boldsymbol{z}^*(t_{N-1})\} \tag{2}$$

that satisfies at any time t_i, $i = 1, \ldots, N$ *the balance equation*

$$\sum_{k=1}^{n} z_k(t_{i-1})\, p_k(t_i) = \sum_{k=1}^{n} z_k(t_i)\, p_k(t_i) + \alpha \sum_{k=1}^{n} |z_k(t_i) - z_k(t_{i-1})| p_k(t_i) \tag{3}$$

such that

$$\langle \boldsymbol{z}^*(t_{N-1}), \boldsymbol{p}(t_N)\rangle = \max_{\boldsymbol{Z}_{[t_0,t_N]}} \langle \boldsymbol{z}(t_{N-1}), \boldsymbol{p}(t_N)\rangle, \tag{4}$$

where $\langle\ ,\ \rangle$ is the standard scalar product in $\mathbb{R}^n$.

We need the following notation to describe the optimal strategy construction scheme:

$$c_k(t_i, t_{i+1}) = \frac{p_k(t_{i+1})}{p_k(t_i)},\ k = \overline{1,n},\ i = \overline{0,N-1},$$

$$c_{\max}(t_i, t_{i+1}) = \max_{1\le k\le n} c_k(t_i, t_{i+1}),\ k = \overline{1,n},\ i = \overline{0,N-1},$$

$$S(t_i) = \sum_{k=1}^{n} z_k(t_i) p_k(t_i),\ i = \overline{0,N},$$

$$\lambda_k(t_i) = \frac{S_k(t_N)}{S_k(t_i)},\ k = \overline{1,n},\ i = \overline{0,N},$$

$$\omega_k(t_i, t_{i+1}) = \lambda_k(t_{i+1}) c_k(t_i, t_{i+1}),\ k = \overline{1,n},\ i = \overline{0,N-1},$$

$$\omega_{\max}(t_i, t_{i+1}) = \max_{1\le k\le n} \omega_k(t_i, t_{i+1}),\ i = \overline{0,N-1},$$

$$\rho_k(t_i, t_{i+}) = a_\alpha \omega_{\max}(t_i, t_{i+1}) - \omega_k(t_i, t_{i+1}),\ k = \overline{1,n},\ i = \overline{0,N-1},$$

where $a_\alpha = (1-\alpha)(1+\alpha)$.

The optimal strategy is constructed in the direction from the end of the investment horizon to its beginning:

1. For each t_i, select the asset A_m with $\omega_m(t_i, t_{i+1}) = \omega_{\max}(t_i, t_{i+1})$.

2. For each $k = \overline{1,n}$ calculate $\rho_k(t_i, t_{i+1})$.

2.1. If $\rho_k(t_i, t_{i+1}) \le 0$ then the position $z_k(t_{i-1})$ of A_k is not changed under the transition $\boldsymbol{z}(t_{i-1}) \to \boldsymbol{z}(t_i)$. In that case set

$$\lambda_k(t_i) = \omega_k(t_i, t_{i+1}) = c_k(t_i, t_{i+1})\, \lambda_k(t_{i+1}).$$

2.2. If $\rho_k(t_i, t_{i+1}) > 0$ then sell all units of the asset A_k and invest the money from its sale in the asset A_m. In that case set

$$\lambda_k(t_i) = a_\alpha\, \omega_{\max}(t_i, t_{i+1}).$$

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Limit theorems connected with cells from a pointed set

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Let n, N be integer numbers. The term *homogeneous allocation scheme of n distinguishable particles by N different cells* will be used for the random variables $\eta_1, \ldots, \eta_N$ with the joint distribution

$$\mathbf{P}\{\eta_1 = k_1, \ldots, \eta_N = k_N\} = \frac{n!}{k_1!\, k_2! \cdots k_N!} \left(\frac{1}{N}\right)^n,$$

where $k_1, k_2, \ldots, k_N$ are nonnegative integer numbers such that $k_1 + k_2 + \ldots + k_N = n$. Denote $\alpha = \frac{n}{N}$.

Let K be an integer number such that $0 < K \le N$. Let r be a nonnegative integer number. We will consider the random variable

$$\mu_r(n, K, N) = \sum_{i=1}^{K} I_{\{\eta_i = r\}}.$$

Observe that $\mu_r(n, K, N)$ is a number of cells from the first K cells which contain r particles.

Theorem 1. *Let $2 \le r$ and N, K, $n \to \infty$ so that $K\, p_r \to \lambda$, where $0 < \lambda < \infty$ and $\frac{K}{N} < C$, $0 < C < 1$. We will suppose that*

$$e\,\alpha - r < C_1, \quad \textit{where} \quad C_1 < \infty, \quad \textit{and} \quad \frac{\alpha}{N} \to 0.$$

Then we have

$$\mathbf{P}\{\mu_r(n, K, N) = k\} = e^{-\lambda}\,\frac{\lambda^k}{k!}\,\big(1 + o(1)\big), \quad k = 0, 1, \dots$$

Consider the random variable

$$\eta_{(K,N)} = \max_{1 \le i \le K} \eta_i.$$

Observe that $\eta_{(K,N)}$ is a maximal value of a cell from the first K cells.

Theorem 2. *Let $r \ge 3$. Suppose that N, K, $n \to \infty$ so that*

$$\frac{\alpha}{r} \to 0, \quad \frac{\alpha}{N} \to 0, \quad K\, p_{r+1} \to \lambda, \quad \frac{K}{N} < C,$$

where $C < 1$ and $0 < \lambda < \infty$. Then we have

$$\mathbf{P}\{\eta_{(K,N)} = r\} = e^{-\lambda} + o(1), \quad \mathbf{P}\{\eta_{(K,N)} = r + 1\} = 1 - e^{-\lambda} + o(1).$$

Consider the random variable

$$\eta_{(K,1)} = \min_{1 \le i \le K} \eta_i.$$

Observe that $\eta_{(K,1)}$ is a minimal value of a cell from the first K cells.

Theorem 3. *Suppose $r \ge 3$, N, K, $n \to \infty$ so that*

$$\frac{\alpha}{r} \to \infty, \quad \frac{\alpha}{N} \to 0, \quad K\, p_{r-1} \to \lambda, \quad \frac{K}{N} < C,$$

where $0 < \lambda < \infty$, $0 < C < 1$. *Then we have*

$$\mathbf{P}\{\eta_{(K,1)} = r-1\} = 1 - e^{-\lambda} + o(1), \quad \mathbf{P}\{\eta_{(K,1)} = r\} = e^{-\lambda} + o(1).$$

Remark 1. Limit theorems for $\mu_r(n,K,N)$, $\eta_{(K,N)}$, $\eta_{(K,1)}$ for the case $K = N$ were obtained in many paper (see [1] and the bibliography therein). In [2] limit theorems were obtained for $\mu_0(n,K,N)$.

Remark 2. Let $A \subset \{1,2,\dots,N\}$, $|A| = K$. Denote

$$\mu_r(n,A,N) = \sum_{i\in A} I_{\{\eta_i = r\}}, \quad \eta_{(A,N)} = \max_{i\in A} \eta_i, \quad \eta_{(A,1)} = \min_{i\in A} \eta_i.$$

The distributions of $\mu_r(n,A,N)$, $\eta_{(A,N)}$ and $\eta_{(A,1)}$ coincide with the distributions of $\mu_r(n,K,N)$, $\eta_{(K,N)}$ and $\eta_{(K,1)}$, correspondingly. So Theorem 1, Theorem 2, Theorem 3 can be considered as theorems for $\mu_r(n,A,N)$, $\eta_{(A,N)}$ and $\eta_{(A,1)}$.

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Infinity-channel queueing system with time-depending intensity of input. Fast and slow growth of the intensity

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The operation of various queueing systems is of increasing interest when the intensity of the incoming flow is high. Therefore, a lot of papers is devoted to the consideration of certain systems in conditions of increasing intensity of the incoming flow. In the report, we consider an infinite-channel system with heavy tails of service times. The property of gravity of tails leads to the fact that, unlike systems with light tails

of maintenance times, two different situations are possible. Namely, depending on whether the intensity grows slowly or rapidly in comparison with the rate of decrease of the tails of service times, convergence to a stable law turns out, or, under a different normalizing factor, to a normal law. The report is a generalization of some results of paper of Mikosch etc. [1], in which for a system with a Poisson input flow with constant intensity the convergence of finite-dimensional distributions is proved.

Consider a Poisson flow on $\mathbb{R}$ with periodic and integrable over the period intensity $\lambda(t)$. Let τ denotes the period of $\lambda(t)$ and $(\Gamma_k, -\infty < k < \infty)$ be points of this Poisson flow (such that $\Gamma_0 < 0 < \Gamma_1$). It is assumed additionally that for any $t \in \mathbb{R}$

$$0 < \lambda_* \leq \lambda(t) \leq \lambda^* < \infty.$$

Denote

$$\Lambda(t) = \int_0^t \lambda(y)dy, \quad \overline{\lambda} = \lim_{t\to\infty} \frac{\Lambda(t)}{t} = \frac{\Lambda(\tau)}{\tau}.$$

We consider a queueing system with infinite number of servers. At every moment Γ_k a claim enters the system and then it is serving during the time interval X_k. We assume $X, X_1, X_2, ...$ to be independent and independent of input flow, identically distributed random variables and

$$P(X > x) = \overline{F}(x) = x^{-\alpha}L(x), \quad x > 0, \quad 1 < \alpha < 2,$$

where $L(x)$ is a slowly varying function.

Let

$$\mu := \mathsf{E}X.$$

Now we introduce a scale parameter T, i.e., consider a family of Poisson flows depending on T, in such a way that

$$\lambda_T(t) = \lambda(t)T^\beta, \quad \beta > 0$$

for $t \in \mathbb{R}$.

Let $N_T(t)$ be a number of claims in the system at time t:

$$N_T(t) = \sum_{k=-\infty}^{\infty} \mathbb{I}_{[\Gamma_k \leq t < \Gamma_k + X_k]}$$

We investigate the total cumulative input $A_T(t)$

$$A(t) = A_T(t) = \int_0^t N_T(s)ds.$$

As $T \to \infty$ then $A_T(t)$ tends to infinity too. We will prove that cumulative input can be approximated by stable law, when the connection rate is slow. Besides, this approximation does not depend on periodicity of $\lambda(t)$ and can be expressed in terms of the average value of $\lambda(t)$ over period. On the other hand, for the case of fast growth of the intensity we need another normalizing coefficient and we show that periodicity of the intensity function plays an essential role and in this case we have for any point $z \in [0, \tau)$ its own normal law as a limit.

Introduce a quantile function

$$b(t) = \left(\frac{1}{\overline{F}}\right)^{\leftarrow}(t),$$

where

$$g^{\leftarrow}(y) = \inf\{x : g(x) \geq y\}.$$

$b(t)$ is a regularly varying function with parameter $\frac{1}{\alpha}$.

We say that the Fast Growth Condition is fulfilled if

$$\lim_{T\to\infty} \frac{b(\overline{\lambda_T}T)}{T} = \infty.$$

We say that the Slow Growth Condition is fulfilled if

$$\lim_{T\to\infty} \frac{b(\overline{\lambda_T}T)}{T} = 0.$$

Remarks. Fast Growth Condition is fulfilled if $\beta > \alpha$. Slow Growth Condition is fulfilled if $\beta < \alpha$.

Introduce

$$A^*(T) = \frac{A(T) - \mu\Lambda(T)}{\sqrt{\overline{\lambda}(T)T^3\overline{F}(T)}}.$$

Theorem. It follows from Fast Growth Condition that for any $z \in [0, \tau]$

$$A^*(n\tau + z) \xrightarrow{d} N(0, \sigma^2(z)) \text{if } n \to \infty,$$

where

$$\sigma^2(z) = \sigma_1^2 + \sigma_2^2(z) + \sigma_3^2,$$
$$\sigma_1^2 = \frac{\alpha}{(2-\alpha)(3-\alpha)},$$

$$\sigma_2^2(z) = \frac{\overline{\lambda}}{(3-\alpha)\int\limits_0^\infty \overline{F}(u)\,\lambda(z-u)\,du},$$

$$\sigma_3^2 = \frac{\overline{\lambda}}{(3-\alpha)\int\limits_0^\infty \overline{F}(u)\,\lambda(-u)\,du}.$$

Theorem. It follows from Slow Growth Condition that

$$\lim_{t\to\infty} \frac{A(T)-\mu\Lambda(T)}{b(\overline{\lambda}T)} \xrightarrow{d} X_\alpha,$$

where

$$\mathsf{E}e^{i\theta X_\alpha} = \exp\big\{-|\theta|^\alpha\big(1 - i\cdot\mathrm{sign}(\theta)\ \mathrm{tg}\,\frac{\pi\alpha}{2}\big)\big\},$$

i.e., X_α has α-stable distribution.

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On the deficiency concept in statistical problems based on the samples with random sizes

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1. Introduction and summary

An interesting quantitative comparison can be obtained by taking a viewpoint similar to that of the asymptotic relative efficiency (ARE) of estimators, and asking for the number $m(n)$ of observations needed by estimator $\delta_{m(n)}(X_1,\ldots,X_{m(n)})$ to match the performance of $\delta_n^*(X_1,\ldots,X_n)$ (based on n observations). Although the difference $m(n)-n$ seems to be a very natural quantity to examine, historically

the ratio $n/m(n)$ was preffered by almost all authors in view of its simpler behaviour. The first general investigation of $m(n) - n$ was carried out by Hodges and Lehmann [1]. They name $m(n) - n$ the deficiency of δ_n with respect to δ_n^* and denote it as

$$d_n = m(n) - n. \tag{1.1}$$

If $\lim_{n\to\infty} d_n$ exists, it is called the asymptotic deficiency of δ_n with respect to δ_n^* and denote as d. At points where no confusion is likely, we shall simply call d the deficiency of δ_n with respect to δ_n^*.

The deficiency of δ_n relative to δ_n^* will then indicate how many observations one loses by insisting on δ_n, and thereby provides a basis for deciding whether or not the price is too high. If the risk functions of these two estimators are

$$R_n(\theta) = \mathsf{E}_\theta\big(\delta_n - g(\theta)\big)^2, \quad R_n^*(\theta) = \mathsf{E}_\theta\big(\delta_n^* - g(\theta)\big)^2,$$

then by definition, $d_n(\theta) \equiv d_n = m(n) - n$, for each n, may be found from

$$R_n^*(\theta) = R_{m(n)}(\theta). \tag{1.2}$$

In order to solve (1.1), $m(n)$ has to be treated as a continuous variable (see [1]). Generally $R_n^*(\theta)$ and $R_n(\theta)$ are not known exactly and we have to use approximations. Here these are obtained by observing that $R_n^*(\theta)$ and $R_n(\theta)$ will typically satisfy asymptotic expansions (a.e.) of the form

$$R_n^* = \frac{a(\theta)}{n^r} + \frac{b(\theta)}{n^{r+s}} + o\big(n^{-(r+s)}\big), \tag{1.3}$$

$$R_n = \frac{a(\theta)}{n^r} + \frac{c(\theta)}{n^{r+s}} + o\big(n^{-(r+s)}\big), \tag{1.4}$$

for certain $a(\theta)$, $b(\theta)$ and $c(\theta)$ not depending on n and certain constants $r > 0$, $s > 0$. The leading term in both expansions is the same in view of the fact that ARE is equal to one. From (1.1)–(1.4) is now easily follows that (see [1])

$$d_n(\theta) \equiv \frac{c(\theta) - b(\theta)}{r\,a(\theta)} n^{(1-s)} + o\big(n^{(1-s)}\big). \tag{1.5}$$

Hence

$$d(\theta) \equiv d = \begin{cases} \pm\infty, & 0 < s < 1, \\ \dfrac{c(\theta) - b(\theta)}{r\,a(\theta)}, & s = 1, \\ 0, & s > 1. \end{cases} \tag{1.6}$$

A useful property of deficiencies is the following (transitivity): if a third estimator $\bar{\delta}_n$ is given, for which the risk $\bar{R}_n(\theta)$ also has an expansion of the form (1.4), the deficiency d of $\bar{\delta}_n$ with respect to δ_n^* satisfies

$$d = d_1 + d_2,$$

where d_1 is the deficiency of $\bar{\delta}_n$ with respect to δ_n and d_2 is the deficiency of δ_n with respect to δ_n^*.

The situation where $s = 1$ seems to be the most interesting one. Hodges nad Lehmann [1] demonstrate the use of deficiency in a number of simple examples for which this is the case (see also [3]).

In the communication, we discuss the number of applications of the deficiency concept in the problems of point estimation and testing statistical hypotheses in the case when number of observations is random.

2. Estimators based on the sample with random size

Consider random variables (r.v.'s) N_1, N_2, ... and $X_1, X_2, ...$, defined on the same probability space $(\Omega, \mathcal{A}, \mathsf{P})$. By X_1, X_2, ..., X_n we will mean statistical observations whereas the r.v. N_n will be regarded as the random sample size depending on the parameter $n \in \mathbb{N}$. Assume that for each $n \geq 1$ the r.v. N_n takes only natural values (i.e., $N_n \in \mathbb{N}$) and is independent of the sequence X_1, X_2, ... Everywhere in what follows the r.v.'s X_1, X_2, ... are assumed independent and identically distributed with distribution depending on $\theta \in \Theta \in \mathbb{R}$.

For every $n \geq 1$ by $T_n = T_n(X_1, ..., X_n)$ denote a statistic, i.e., a real-valued measurable function of X_1, ..., X_n. For each $n \geq 1$ we define a r.v. T_{N_n} by setting $T_{N_n}(\omega) \equiv T_{N_n(\omega)}(X_1(\omega), ..., X_{N_n(\omega)}(\omega))$, $\omega \in \Omega$.

Theorem 2.1.

1. *If* $\delta_n = \delta_n(X_1, \ldots, X_n)$ *is any unbiased estimator of* $g(\theta)$ *that is, it satisfies*

$$\mathsf{E}_\theta \delta_n = g(\theta), \quad \theta \in \Theta$$

and $\delta_{N_n} \equiv \delta_{N_n}(X_1, \ldots, X_{N_n})$, *then*

$$\mathsf{E}_\theta \delta_{N_n} = g(\theta), \quad \theta \in \Theta.$$

2. *Suppose that numbers* $a(\theta)$, $b(\theta)$ *and* $C(\theta) > 0$, $\alpha > 0$, $r > 0$, $s > 0$ *exist such that*

$$\left| R_n^*(\theta) - \frac{a(\theta)}{n^r} - \frac{b(\theta)}{n^{r+s}} \right| \leqslant \frac{C(\theta)}{n^{r+s+\alpha}},$$

where

$$R_n^*(\theta) = \mathsf{E}_\theta \big(\delta_n(X_1, \ldots, X_n) - g(\theta)\big)^2,$$

then

$$\left|R_n(\theta) - a(\theta)\,\mathsf{E}\,N_n^{-r} - b(\theta)\,\mathsf{E}\,N_n^{-r-s}\right| \leqslant C(\theta)\,\mathsf{E}\,N_n^{-r-s-\alpha},$$

where

$$R_n(\theta) = \mathsf{E}_\theta\big(\delta_{N_n}(X_1,\ldots,X_{N_n}) - g(\theta)\big)^2.$$

Corollary 2.1. *Suppose that numbers $a(\theta)$, $b(\theta)$ and $r > 0$, $s > 0$ exist such that*

$$R_n^*(\theta) \equiv \mathsf{E}_\theta\big(\delta_n(X_1,\ldots,X_n) - g(\theta)\big)^2 = \frac{a(\theta)}{n^r} + \frac{b(\theta)}{n^{r+s}},$$

then

$$R_n(\theta) \equiv \mathsf{E}_\theta\big(\delta_{N_n}(X_1,\ldots,X_{N_n}) - g(\theta)\big)^2 = a(\theta)\,\mathsf{E}\,N_n^{-r} + b(\theta)\,\mathsf{E}\,N_n^{-r-s}.$$

Let observations $X_1,\ldots,X_n$ have expectation $\mathsf{E}_\theta\,X_1 = g(\theta)$ and variance $\mathsf{D}_\theta\,X_1 = \sigma^2(\theta)$. The customary estimator for $g(\theta)$ based on n observation is

$$\delta_n = \frac{1}{n}\sum_{i=1}^{n} X_i. \tag{2.1}$$

This estimator is unbiased and consistent, and its variance is

$$R_n^*(\theta) = \mathsf{D}_\theta\,\delta_n = \frac{\sigma^2(\theta)}{n}. \tag{2.2}$$

If this estimator based on the sample with random size we have (see Corollary 2.1)

$$R_n(\theta) = \mathsf{D}_\theta\,\delta_{N_n}(X_1,\ldots,X_{N_n}) = \sigma^2(\theta)\,\mathsf{E}\,N_n^{-1}. \tag{2.3}$$

If $g(\theta)$ is given, we consider the estimator for $\sigma^2(\theta)$ in the form

$$\bar{\delta}_n = \frac{1}{n}\sum_{i=1}^{n}(X_i - g(\theta))^2. \tag{2.4}$$

This estimator is unbiased and consistent, and its variance is

$$\bar{R}_n^*(\theta) = \mathsf{D}_\theta\,\bar{\delta}_n = \frac{\mu_4(\theta) - \sigma^4(\theta)}{n}, \quad \mu_4(\theta) = \mathsf{E}_\theta\,(X_1 - g(\theta))^4. \tag{2.5}$$

For this estimator with random size one have

$$\bar{R}_n(\theta) = \mathsf{D}_\theta\, \bar{\delta}_{N_n}(X_1, \ldots, X_n) = \big(\mu_4(\theta) - \sigma^4(\theta)\big)\, \mathsf{E}\, N_n^{-1}. \tag{2.6}$$

In the preceding example, suppose that $g(\theta)$ is unknown but that instead of (2.4) we are willing to consider any estimator of the form (see (2.1))

$$\tilde{\delta}_n^{(\gamma)} \equiv \tilde{\delta}_n = \frac{1}{n+\gamma} \sum_{i=1}^{n} \big(X_i - \delta_n\big)^2, \quad \gamma \in \mathbb{R}. \tag{2.7}$$

If $\gamma \neq -1$, this will not be unbiased but may have a smaller expected squared error that the unbiased estimator with $\gamma = -1$.

One easily find (see [1], (3.6) and [2])

$$\tilde{R}_n^*(\theta) = \sigma^4(\theta) \Big[\frac{\mu_4(\theta)/\sigma^4(\theta) - 1}{n} +$$

$$+ \frac{(\gamma+1)^2 + 2 - 2\,(\gamma+1)\,(\mu_4(\theta)/\sigma^4(\theta) - 1)}{n^2} \Big] + O\big(n^{-3}\big). \tag{2.8}$$

Using Theorem 1.1, we have

$$\tilde{R}_n(\theta) = \mathsf{E}_\theta \left(\tilde{\delta}_{N_n}(X_1, \ldots, X_{N_n}) - \sigma^2(\theta)\right)^2 =$$

$$= \sigma^4(\theta) \Big[(\mu_4(\theta)/\sigma^4(\theta) - 1)\, \mathsf{E}\, N_n^{-1} +$$

$$+ \big\{(\gamma+1)^2 + 2 - 2\,(\gamma+1)\,(\mu_4(\theta)/\sigma^4(\theta) - 1)\big\}\, \mathsf{E}\, N_n^{-2} \Big] + O\big(\mathsf{E}\, N_n^{-3}\big). \tag{2.9}$$

3. Deficiencies of some estimators based on the samples with random size

When the deficiencies of statistical estimators constructed from samples of random size $N_{m(n)}$ and the corresponding estimators constructed from samples of non-random size n (under the condition $\mathsf{E}\, N_n = n$) are evaluated, we actually compare the expected size $m(n)$ of a random sample with n by virtue of the quantity $d_n = m(n) - n$ and its limit value.

We now apply the results of section 2 to the three examples given in this section. Let M_n be the Poisson r.v. with parameter $n-1$, $n \geqslant 2$, i.e.

$$\mathsf{P}\left(M_n = k\right) = e^{(1-n)}\, \frac{(n-1)^k}{k!}, \quad k = 0, 1, \ldots$$

Define the random size as $N_n = M_n + 1$, then $\mathsf{E}\, N_n = n$ and $\mathsf{E}\, N_n^{-1} = \frac{1}{n} + \frac{1}{n^2} + o\big(n^{-2}\big)$. The deficiency of δ_{N_n} relative to δ_n (see (2.1)) is given by (2.2), (2.3), (3.1) and (1.6) with $r = s = 1$, $a(\theta) = \sigma^2(\theta)$, $b(\theta) = 0$, $c(\theta) = \sigma^4(\theta)$, and hence is equal to $d = 1$. Similarly, the deficiency of $\bar{\delta}_{N_n}$ relative to $\bar{\delta}_n$ (see (2.4)) is given by (2.5), (2.6) and (1.6) with $r = s = 1$, $a(\theta) = c(\theta) = \mu_4(\theta) - \sigma^4(\theta)$, $b(\theta) = 0$, and hence is equal to $\bar{d} = 1$. Consider now third example (see (2.7)). We have $\mathsf{E}\, N_n^{-2} \sim \frac{1}{n^2}$, $n \to \infty$. Now the deficiency of $\tilde{\delta}_{N_n}$ relative to $\tilde{\delta}_n$ (see (2.7)) is given by (2.8), (2.9) and (1.6) with $r = s = 1$ and hence is equal to $\tilde{d} = 1$ and the deficiency of $\tilde{\delta}_{N_n}^{(\gamma_1)}$ relative to $\tilde{\delta}_{N_n}^{(\gamma_2)}$ (see (2.7)) is given by (1.6) with $r = s = 1$ and hence is equal to

$$\tilde{d}_{\gamma_1,\gamma_2} = (\gamma_1 - \gamma_2)\Big(\frac{\gamma_1 + \gamma_2 + 2}{\mu_4(\theta)/\sigma^4(\theta) - 1} - 2\Big).$$

These examples illustrate the following

Theorem 3.1. *Suppose that numbers $a(\theta)$, $b(\theta)$ and k_1, k_2 exist such that*

$$R_n^*(\theta) = \frac{a(\theta)}{n} + \frac{b(\theta)}{n^2} = o\big(n^{-2}\big)$$

and

$$\mathsf{E}\, N_n^{-1} = \frac{1}{n} + \frac{k_1}{n^2} + o\big(n^{-2}\big), \qquad \mathsf{E}\, N_n^{-2} = \frac{k_2}{n^2} + o\big(n^{-2}\big),$$

$$\mathsf{E}\, N_n^{-3} = o\big(n^{-2}\big),$$

then the asymptotic deficiency of $\delta_{N_n}(X_1, \ldots, X_{N_n})$ with respect to $\delta_n(X_1, \ldots, X_n)$ is equal to

$$d(\theta) = \frac{k_1\, a(\theta) + b(\theta)\, k_2 - b(\theta)}{a(\theta)}.$$

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Performance modeling of finite-source retrial queueing systems with collisions and blocking

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In this paper we investigate a single-server retrial queueing system with collision of the customer and an unreliable server (J. S. Kim [2]). The results are provided by the help of software tool MOSEL-2 (T. Bérczes, J. Sztrik, Á. Tóth, A. Nazarov [1]). The number of sources of calls is finite and collision (Nazarov and Kvach and Yampolsky [3], Tóth and Bérczes and Sztrik and Kvach [4]) can take place. If a customer finds the server idle, he enters into service immediately. The failure of server block the system's operation therefore the arriving customers can not enter the system, meaning that those calls are lost. Our interest is to give the main steady-state performance measures of the system computed by the help of the MOSEL-2 tool. Various figures represent the impact of blocking phenomenon on the main performance measures like mean number of customers in the system, mean response time, mean time spent in service, mean waiting time (man time spent in the orbit).

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Complex network models and generalized allocation scheme

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We consider configuration graphs where vertex degrees are independent identically distributed random variables with different probability distributions. Configuration graphs were first introduced by Bollobas in [1]. Such random graphs frequently prove useful as models of complex communication networks like the transport, telephone, electric networks, social relationships and the main global network – Internet (see, e.g., Hofstad [2]). Let N be a number of vertices in the graph. Vertex degrees form semiedges that are numbered in an arbitrary order. If the sum of vertex degrees is odd one extra vertex with degree one is added. The graph is constructed by joining all the semiedges pairwise equiprobably to form edges. Those graphs admit multiple edges and loops. Numerous observations of real networks suggest that the distribution of degree ξ of each vertex can be specified by the relation

$$\mathbf{P}\{\xi \geqslant k\} = \frac{h(k)}{k^{\tau}}, \qquad k = 1, 2, \ldots, \quad \tau > 0, \tag{1}$$

where $h(k)$ is a slowly varying function. Reittu and Norros [3] are sure that the function $h(k)$ in (1) does not influence limit results as $N \to \infty$ and we can replace $h(k)$ with the constant 1. Then

$$\mathbf{P}\{\xi = k\} = k^{-\tau} - (k+1)^{-\tau}, \qquad k = 1, 2, \ldots, \quad \tau > 0. \qquad (2)$$

Recently there appeared some works where the authors note that with the network size growth the vertex degree distributions may change and even become random.

We consider two types of conditional configuration graphs. One of them is a subset of graphs where the sum of vertex degrees is known and it is equal to n. In the other subset the sum of vertex degrees was bounded from above by n. Such conditional graphs can be useful for modeling of networks for which we can estimate the number of links. They are useful also for studying networks without conditions on the number of edges by averaging the results of conditional graphs with respect to the distribution of the sum of degrees. Assume that the parameter τ of distribution (2) depends on N or it is a random variable. For different types of parameter τ behaviour we find the limiting distributions of the maximum vertex degree and of the number of vertices with a given degree for various zones of convergence N and n to infinity (see Pavlov, Cheplyukova [4,5]). The main results of this work are limit theorems for the same degree structure characteristics of conditional configuration graphs when the distribution of ξ is unknown and we can estimate only limit behaviour of the distribution tail. There results were proved using the generalized allocation scheme which was studied by Kolchin [6] and its analogue (Chuprunov and Fazekas [7]). Our theorems can be looked as applications of this scheme in the case of independent random variables with unknown distributions.

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Generalization of the Rao–Robson–Nikulin test

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Let the probability distribution of a random variable ξ be given by the density function $f_0 = f_0[x, \boldsymbol{\theta}]$ with respect to some σ-finite measure ν, which is either a Lebesgue measure or a counting measure, and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_s) \in \boldsymbol{\Theta}_0 \subset \mathbf{R}^s$ is an unknown s-dimensional distribution parameter. There is a sample $\mathbf{X} = (X_1, \ldots, X_n)$, whose elements are independent random variables having the same distribution F_0 as the random variable ξ.

The problem of testing a complex hypothesis $H_0 : \ f_0[x, \boldsymbol{\theta}] \in \mathcal{P} = \{f_0 : f_0 \in \mathbf{f}_0\}$ is considered with the help of the Pearson's chi-squared test. In this case, the hypothesis

$$H_0' : \mathbf{P}\left(\xi \in \Delta_j\right) = \pi_j[\boldsymbol{\theta}],\ \boldsymbol{\theta} \in \boldsymbol{\Theta}_0, \quad j = 1, \ldots, J-1,$$

is usually verified instead of H_0. Here

$$\pi_j[\boldsymbol{\theta}] = \int_{\Delta_j} f_0[x, \boldsymbol{\theta}]\nu[dx] = \mathbb{E}_{F_0}\, I_{\Delta_j}[\xi];$$

$\Delta_1, \ldots, \Delta_J$ are atoms of a partition of the support for the distribution of ξ, $J > s$; $I_B[x]$ is the indicator of the event $x \in B$.

If $\boldsymbol{\theta}$ is the known parameter, then the hypothesis H_0' is verified with the help of statistics

$$X_n^2[\boldsymbol{\theta}] = \sum_{j=1}^{J} \frac{\left(U_j - n\pi_j[\boldsymbol{\theta}]\right)^2}{n\pi_j[\boldsymbol{\theta}]} = \frac{1}{n}\sum_{j=1}^{J} \frac{U_j^2}{\pi_j[\boldsymbol{\theta}]} - n, \tag{1}$$

where U_j is the number of sampling elements belonging to the atom Δ_j.

If instead of the unknown parameter $\boldsymbol{\theta}$, its maximum likelihood estimate $\tilde{\boldsymbol{\theta}}_n$ is used for the sample $\mathbf{X}$, then the Nikulin–Rao–Robson test [1] is used to verify the complex hypothesis H_0'. If

$$\mathbf{G}[\boldsymbol{\theta}] = \mathbf{i}[\boldsymbol{\theta}] - \mathbf{C}\mathbf{C}^\top$$

is a nondegenerate matrix, then the statistics Y_n^2 of this criterion is defined by the equation

$$Y_n^2 = X_n^2[\tilde{\boldsymbol{\theta}}_n] + \tilde{\boldsymbol{\upsilon}}^\top \mathbf{G}^{-1}[\tilde{\boldsymbol{\theta}}_n]\, \tilde{\boldsymbol{\upsilon}}/n, \tag{2}$$

where $\mathbf{i}[\boldsymbol{\theta}]$ is a Fisher's information matrix for X_1, $\tilde{i}_{\ell_1\ell_2}$ is an element of the matrix $\mathbf{i}[\tilde{\boldsymbol{\theta}}_n]$, $\tilde{\boldsymbol{\upsilon}} = (\tilde{\upsilon}_1, \ldots, \tilde{\upsilon}_s)^\top$, $\mathbf{G}[\tilde{\boldsymbol{\theta}}_n] = (\tilde{g}_{\ell_1\ell_2})_{s\times s}$,

$$\mathbf{C} = \left(\frac{1}{\sqrt{\pi_j[\boldsymbol{\theta}]}} \frac{\partial \pi_j[\boldsymbol{\theta}]}{\partial \theta_i} \right)_{s\times J}, \qquad \tilde{\upsilon}_j = \sum_{i=1}^{J} \frac{U_i}{\pi_i[\tilde{\boldsymbol{\theta}}_n]} \frac{\partial \pi_i[\tilde{\boldsymbol{\theta}}_n]}{\partial \theta_j},$$

$$\tilde{g}_{\ell_1\ell_2} = \tilde{i}_{\ell_1\ell_2} - \sum_{j=1}^{J} \frac{1}{\pi_j[\tilde{\boldsymbol{\theta}}_n]} \frac{\partial \pi_j[\tilde{\boldsymbol{\theta}}_n]}{\partial \theta_{\ell_1}} \frac{\partial \pi_j[\tilde{\boldsymbol{\theta}}_n]}{\partial \theta_{\ell_2}}.$$

According to [1, Theorem 2.3], if the certain regularity conditions $\mathbb{A}$ are satisfied and $n \to \infty$, then a sequence of values of statistics (2) converges to a random variable having the chi-square distribution with $J-1$ degrees of freedom χ^2_{J-1}. This result is the basis of the asymptotic *Nikulin-Rao-Robson test*: the hypothesis H_0' should be rejected with an asymptotic significance level α if $Y_n^2 \geq \chi^2_{1-\alpha}[J-1]$, where $\chi^2_{1-\alpha}[J-1]$ is the $(1-\alpha)$-quantile of the distribution χ^2_{J-1}.

A generalization of this criterion on the basis of chi-squared statistics χ_h is proposed below. New criterion allows to verify a more general null hypothesis than H_0'

$$H_0'' : \mathbb{E}_{F_0} h_j[\xi] = \mu_j[\boldsymbol{\theta}],\ j = 1, \ldots, m. \tag{3}$$

Here $\{\mu_j[\boldsymbol{\theta}],\, j = 1, \ldots, m\}$ are the mathematical expectations for a given set of functions $\{h_j[x],\, j = 1, \ldots, m\}$ calculated on the basis of the hypothetical distribution F_0 of the random variable ξ. Note that the null hypothesis H_0' corresponds to the set of indicator functions $h_j[x] = I_{\Delta_j}[x],\ j = 1, \ldots, J-1$.

Limit behavior of the test statistics χ_h is described by the following statement.

Theorem. *Suppose the conditions* $\mathbb{A}$ *are satisfied and the vector function* $\mathbf{h}[x] = (h_1[x], \ldots, h_m[x])^\top$ *has the following properties*:

(i) *the vector function* $\boldsymbol{\mu}[\boldsymbol{\theta}] = (\mu_1[\boldsymbol{\theta}], \ldots, \mu_m[\boldsymbol{\theta}])^\top = \mathbb{E}_{F_0}\mathbf{h}[\xi]$ *is continuously differentiable at the point* $\boldsymbol{\theta}$;

(ii) *the covariance matrix of the vector* $\mathbf{h}[\xi]$, $\boldsymbol{\Sigma}_h[\boldsymbol{\theta}] = \mathbb{V}_{F_0}\mathbf{h}[\xi]$, *and the information matrix* $\mathbf{i}[\boldsymbol{\theta}]$ *are continuous at the point* $\boldsymbol{\theta}$;

(iii) *the matrices* $\boldsymbol{\Sigma}_h[\boldsymbol{\theta}]$ *and* $\boldsymbol{\psi}[\boldsymbol{\theta}] = \mathbf{i}[\boldsymbol{\theta}] - \dot{\boldsymbol{\mu}}^\top[\boldsymbol{\theta}]\,\boldsymbol{\Sigma}_h^{-1}[\boldsymbol{\theta}]\,\dot{\boldsymbol{\mu}}[\boldsymbol{\theta}]$, *where* $\dot{\boldsymbol{\mu}}[\boldsymbol{\theta}] = (\partial\mu_i[\boldsymbol{\theta}]/\partial\theta_j)_{m\times s}$, *are not degenerate.*

Then under the hypothesis H_0'', *the statistics*

$$\chi_h = \frac{1}{n}\left\{(\mathbf{u} - n\tilde{\boldsymbol{\mu}})^\top \boldsymbol{\Sigma}_h^{-1}[\tilde{\boldsymbol{\theta}}_n]\,(\mathbf{u} - n\tilde{\boldsymbol{\mu}}) + \tilde{\mathbf{v}}^\top\boldsymbol{\psi}^{-1}[\tilde{\boldsymbol{\theta}}_n]\tilde{\mathbf{v}}\right\}, \qquad (4)$$

$$\mathbf{u} = \sum_{i=1}^{n}\mathbf{h}[X_i], \quad \tilde{\boldsymbol{\mu}} = \boldsymbol{\mu}[\tilde{\boldsymbol{\theta}}_n], \quad \mathbf{v}[\boldsymbol{\theta}] = \dot{\boldsymbol{\mu}}^\top[\boldsymbol{\theta}]\boldsymbol{\Sigma}_h^{-1}[\boldsymbol{\theta}]\,(\mathbf{u} - n\tilde{\boldsymbol{\mu}}), \quad \tilde{\mathbf{v}} = \mathbf{v}[\tilde{\boldsymbol{\theta}}],$$

converges in distribution to a random variable having a distribution χ^2_m, *when* $n \to \infty$.

This result allows to construct a criterion for testing the hypothesis H_0''.

***Generalized Nikulin–Rao–Robson* test.** Hypothesis H_0'' is rejected with asymptotic level of significance α, if $\chi_h \geq \chi^2_{1-\alpha}[m]$, where $\chi^2_{1-\alpha}[m]$ is the $(1-\alpha)$-quantile of the chi-square distribution with m degrees of freedom.

Explicit formulas for computing the statistics χ_h succeeded to get in the following cases:

$$h_j[x] = \eta_j[x]I_{\Delta_j}[x], \quad j = 1, \ldots, m, \quad m \leq J, \qquad (5)$$

$$h_j[x] = \begin{cases}\eta_j[x]I_{\Delta_j}[x], & \text{при } j = 1, \ldots, J,\\ I_{\Delta_{j-J}}[x], & \text{при } j = J+1, \ldots, m,\end{cases} \quad m - J \leq J - 1, \qquad (6)$$

$$h_j[x] = \begin{cases}\eta_j[x]I_{\Delta_j}[x], & \text{при } j = 1, \ldots, M,\\ I_{\Delta_j}[x], & \text{при } j = M+1, \ldots, J,\\ I_{\Delta_{j-J}}[x], & \text{при } j = J+1, \ldots, m,\end{cases} \quad m - J \leq M - 1, \qquad (7)$$

$$\mathbf{h}[x] = \left(I_{\Delta_1}[x], \ldots, I_{\Delta_{J-1}}[x], h_J[x]\right)^\top. \qquad (8)$$

It was found that one of the terms of each formulas obtained has the form of the right-hand side of equation (1). This fact takes place in the case of statistics (2).

Comment. If weak regularity assumptions are satisfied, then the limit distribution of the statistics χ_h is the same as in the case when the limits of the atoms are not pre-fixed but are chosen as data functions.

In [2]–[4], we have previously considered special cases of constructing chi-squared tests for checking null hypothesis for the one-parameter form H_0''.

Comparison of the power of the generalized Nikulin–Rao–Robson test for $h_j[x] = x\, I_{\Delta_j}[x], \quad j = 1, \ldots, J-1$, with the power of other two chi-square tests for checking the hypothesis about the normal distribution law of the random variable $\xi \sim \mathcal{N}(\mu;\, \sigma^2)$, will be fulfilled in the final part of our submission. The power values for other two tests can be found in [5]. As alternative distributions, we consider the logistic and generalized normal distributions with 4 as the form parameter. The reason is that these distributions are the nearest ones to the given normal distribution. The appropriate probability density functions are defined as follows:

$$f_1[x] = \frac{1}{\beta_1} \frac{\exp\left[-\dfrac{x-\mu}{\beta_1}\right]}{\left(1 + \exp\left[-\dfrac{x-\mu}{\beta_1}\right]\right)^2}, \quad \beta_1 = \sigma\frac{\sqrt{3}}{\pi},$$

$$f_2[x] = \frac{\sqrt{2}}{\beta_2 \Gamma[1/4]} \exp\left[-\frac{(x-\mu)^4}{4\beta_2^4}\right], \quad \beta_2 = \sigma\sqrt{\frac{\Gamma[1/4]}{2\Gamma[3/4]}}.$$

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Optimal control of a single-server retrial queue with two-phase service

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The retrial queuing model with a single server providing two phases of service has many applications and has received significant attention in the literature. One can find important applications of the two-phase service models in multimedia communications, packet transmissions, production lines and telecommunication systems. The paper of I. Dimitriou & C. Langaris [2] presented an analysis of a retrial queue with two phases of service and server vacation. Every customer was placed in a single queue while waiting to be served. When a customer finished the first stage of service, then, he either went to the second phase with probability $1-p$ or, with probability p, departed and joined a retrial box from which he repeated the demand for the second phase, and left the system after service completion.

In this paper, we consider the problem of a dynamic routing control retrial queue with a single server providing two phases of service. Customers arrive to the system according to a Poisson process with parameter λ. The service for each customer consists of 2 independent phases d_1 and d_2, each of which has an exponential distribution with mean $1/\mu$. Every customer must receive service in two phases before leaving the system. Arriving customers join a single ordinary queue and wait to start their service sequences in the first phase. At the end of the first phase d_1, the server may start computing the second phase d_2 for the same customer or stop the actual service sequence in phase 1 and place the customer in the retrial box. In the latter case, the server immediately serves the next customer in the first phase. The customers in the retrial box make a service request with respect to a Poisson process with rate θ and can receive the second phase d_2 only when the server is idle. The holding cost per customer per unit time in the primary queue is c_1 and the retrial box is c_2. The goal is to find a routing policy that minimizes the expected total discounted holding cost over infinite horizon.

The problem of routing control in the retrial queue can be formulated as a semi-Markov decision process in which the decision epoch is the service completion point of the first stage d_1. At a decision epoch, the system controller must decide whether to keep the customer in service or route him to the retrial box. When in service, the customer receives service d_2 and then, leaves the system. When in the retrial box, the customer waits for a random amount of time and then, tries to find the server available again to complete the second phase of service d_2 and leaves the system. The decision is taken according to the state of the system and the cost induced by this state and it is based essentially on minimizing the cost of waiting in the system.

State definitions. We note that the presence of i customers in the primary queue means that there are $i-1$ customers on hold and one customer in service. When a customer enters service, the 2 associated phases are served one at a time; thus, i customers in the primary queue corresponds to $2\,i$ (or $2\,i-1$) phases (see Tijms [8], chap.2, p.104). Consequently, we can distinguish between states $(2\,i, j)$ and $(2\,i-1, j)$, where $2\,i$, $(2\,i-1))$ denotes the number of service phases yet to be completed and j denotes the number of customers in the retrial box. Finally, the state of the server is described as odd or even using the symbols ($2\,i$ and $2\,i-1$).

Therefore, the state space of the retrial queuing system is

$$S = \{(2i, j), ((2i-1)^+, j) \ / \ i, j = 0, 1, 2, ...\}.$$

Under some regularity conditions, we prove the existence of an optimal policy that minimizes the expected total discounted cost of the system. In the case of socially optimal routing policies, we show that such a policy is described by a switching curve based on the number of customers in the system.

We conjecture that the optimal threshold is a non-decreasing function of the number of customers in the retrial box. The structure of our switching curve is shown in Fig. 1. Numerical results for the optimal threshold for different parameter values are provided and confirm the validity of this result.

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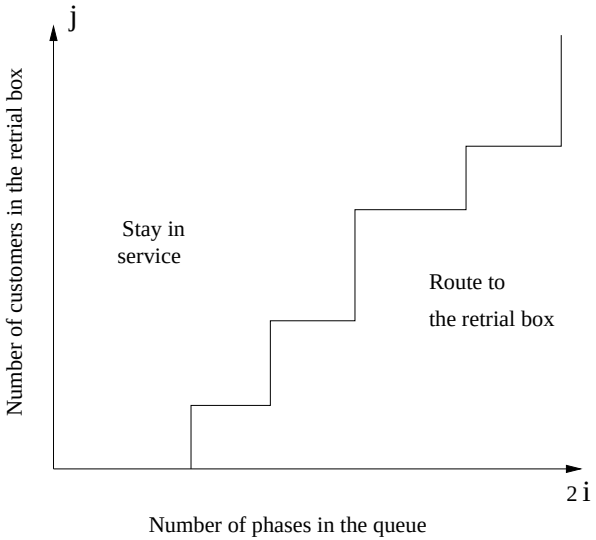

Fig. 1: The structure of our switching curve

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Structural improvement of Esseen's and Rozovskii's inequalities

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Let $X_1, \ldots, X_n$ be independent random variables on a common probability space $(\Omega, \mathcal{F}, \mathsf{P})$ with $\mathsf{E}X_k = 0$, $\mathsf{E}X_k^2 = \sigma_k^2 < \infty$ and $B_n^2 = \sum_{k=1}^n \sigma_k^2 > 0$. Denote

$$\sigma_k^2(z) = \mathsf{E}X_k^2 1(|X_k| \geqslant z), \quad L_n(z) = \frac{1}{B_n^2}\sum_{k=1}^n \sigma_k^2(zB_n), \quad z > 0,$$

$$\mu_k(z) = \mathsf{E}X_k^3 1\left(|X_k| < z\right), \quad M_n(z) = \frac{1}{B_n^3}\sum_{k=1}^n \mu_k(zB_n), \quad z > 0,$$

$$F_n(x) = \mathsf{P}(X_1 + \ldots + X_n < xB_n), \quad \Phi(x) = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-t^2/2}dt, \quad x \in \mathbf{R},$$

$$\Delta_n = \Delta_n(F_1, \ldots, F_n) = \sup_{x\in\mathbf{R}} |F_n(x) - \Phi(x)|.$$

For every $\varepsilon > 0, \gamma > 0$ we prove inequalities

$$\Delta_n \leqslant C_E \cdot L_{E,n}^3(\varepsilon, \gamma), \quad L_{E,n}^3(\varepsilon, \gamma) := \sup_{0<z\leqslant\varepsilon} \{\gamma|M_n(z)| + zL_n(z)\}, \quad (1)$$

$$\Delta_n \leqslant C_R \cdot L_{R,n}^3(\varepsilon, \gamma), \quad L_{R,n}^3(\varepsilon, \gamma) := \big(\gamma|M_n(\varepsilon)| + \sup_{0<z\leqslant\varepsilon} zL_n(z)\big), \quad (2)$$

where constants $C_E = C_E(\varepsilon, \gamma), C_R = C_R(\varepsilon, \gamma)$ depend only on ε, γ.

These inequalities improve and generalize Esseen's and Rozovskii's results [1],[2] and, according to Zolotarev's [3] classification, can be called *natural* convergence rate estimates in the Lindeberg–Feller theorem.

Similary to Kolmogorov [4], where the classical Berry–Esseen inequality was discussed, we also introduce the so-called *asymptotically exact constants* in (1), (2)

$$C_E^*(\varepsilon, \gamma) = \limsup_{\ell\to 0} \sup_{n, F_1, \ldots, F_n} \left\{\Delta_n(F_1, \ldots, F_n)/\ell : L_{E,n}^3(\varepsilon, \gamma) = \ell\right\},$$

$$C_R^*(\varepsilon, \gamma) = \limsup_{\ell\to 0} \sup_{n, F_1, \ldots, F_n} \left\{\Delta_n(F_1, \ldots, F_n)/\ell : L_{R,n}^3(\varepsilon, \gamma) = \ell\right\},$$

and present their upper bounds for every $\varepsilon > 0$ and $\gamma > 0$.

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Uniform limit theorems for censored integrals with application in estimation theory

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Following [1–3], consider competing risks model, where we are interested in observing of random variable (r.v.) Z with distribution function (d.f.) H and pairwise disjoint events $\{A^{(i)},\, i \in J = \{1, ..., k\}\}$, such that

$$\mathbf{P}\Big(\bigcup_{i=1}^{k} A^{(i)}\Big) = 1$$

(see, [3]). In fact, we are interested in joint properties of pairs $\{\big(Z, A^{(i)}\big),\, i \in J\}$.

Let's introduce subdistribution functions

$$\Big\{H(x;i) = \mathbf{P}\big(Z \le x, A^{(i)}\big), \quad (x;i) \in \mathbb{R} \times J\Big\},$$

for which $H(x;1) + ... + H(x;k) = H(x)$. Here we suppose that the pairs $\{\big(Z, A^{(i)}\big),\, i \in J\}$ are censored from right and left by r.v.-s Y

and L with corresponding d.f.-s G and K and that r.v.-s $\{Z, Y, L\}$ are independent.

Observation is available the sample

$$S^{(n)} = \left\{ \left(\zeta_j; \chi_j^{(0)}; \chi_j^{(1)}, ..., \chi_j^{(k)} \right), \quad j = 1, ..., n \right\},$$

where

$$\zeta_j = \max\left\{ L_j, \min\{Z_j, Y_j\} \right\}, \quad \chi_j^{(i)} = I\left(D_j^{(i)}\right), \quad i \in \overline{J} = J \cup \{0\},$$
$$D_j^{(0)} = \left\{ \min\{Z_j, Y_j\} < L_j \right\} \cup \{L_j \le Y_j < Z_j\},$$
$$D_j^{(i)} = A_j^{(i)} \cap \{L_j \le Z_j \le Y_j\}, \quad i \in J$$

and $\left\{ Z_j, L_j, D_j^{(0)}, D_j^{(1)}, ..., D_j^{(k)}, \ j \ge 1 \right\}$ consequence of independent and identically distributed copies of aggregate $\left\{ Z, L, Y, D^{(0)}, D^{(1)}, ..., D^{(k)} \right\}$. It is not difficult to see that d.f. of r.v. $\zeta = \max\left\{ L, \min\{Z, Y\} \right\}$ is

$$E(x) = \mathbf{P}(\zeta \le x) = K(x) \left[1 - \left(1 - G(x) \right) \left(1 - H(x) \right) \right].$$

Note that in the sample $S^{(n)}$ random pairs $\left(Z_j, A_j^{(i)} \right)$ observable only in the case of $\chi_j^{(i)} = 1$, $i \in J$.

Consider survival functionals (exponentional-hazard functions)

$$1 - F_\tau(x; i) = \exp\left[- \Lambda_\tau(x; i) \right], \quad i \in J$$

and their estimators

$$1 - F_{n\tau}(x; i) = \exp\left[- \Lambda_{n\tau}(x; i) \right], \quad i \in J,$$

where

$$\Lambda_\tau(x; i) = \int_{[\tau; x]} \frac{dH(u; i)}{1 - H(u-)}, \quad (x; i) \in [\tau, \infty) \times J,$$

$$\Lambda_{n\tau}(x; i) = \frac{1}{n} \sum_{j=1}^{n} \frac{I(\zeta_j \le x, \chi_j^{(i)} = 1)}{q_n(\zeta_j)}, \quad q_n(x) = K_n(x) - \Delta T_{1n}(x; 0),$$

$$T_{1n}(x; 0) = \frac{1}{n} \sum_{j=1}^{n} I\left(\zeta_j \le x, \chi_j^{(0)} = 1 \right), \quad E_n(x) = \frac{1}{n} \sum_{j=1}^{n} I(\zeta_j \le x),$$

$$K_n(x) = \exp\left[- \int_{[x, \infty)} \frac{dT_{1n}(u; 0)}{E_n(u-)} \right].$$

Let $\mathcal{L}_q(Q)$ be the space of functions $f : \mathbb{R} \to \mathbb{R}$ with the norm

$$\|f\|_{Q,q} = \Big\{ \int_{\mathbb{R}} |f|^q \, dQ \Big\}^{1/q}, \quad where \quad Q(x) = \int_{\tau}^{x} \frac{dE(u)}{[K(u)(1-\gamma(u-))]^2}.$$

We introduce some notations from metric entropy theory in [4] adapting to considered competing risks model.

Let $h_q(\varepsilon) = \log N_{[]}\big(\varepsilon, \mathcal{F}, \mathcal{L}_q(Q)\big)$ be the metric entropy with the bracketing number $N_{[]}\big(\varepsilon, \mathcal{F}, \mathcal{L}_q(Q)\big)$ of the class $\mathcal{F}$ in $\mathcal{L}_q(Q)$. We define also the integral of the metric entropy with bracketing as

$$J_{[]}^{(q)}(\delta) = J_{[]}\big(\delta; \mathcal{F}; \mathcal{L}_q(Q)\big) = \int_0^{\delta} \big[h_q(\varepsilon)\big]^{1/2} \, d\varepsilon, \quad 0 < \delta \le 1.$$

Introduce $\mathcal{F}$-indexed process for each $i \in J$ as

$$G_n^{(i)} f = \int_{\tau}^{T} f(x) \, d\big(F_{n\tau}(x;i) - F_{\tau}(x;i)\big), \qquad f \in \mathcal{F},$$

where $\tau < T < T_Q = \sup[x : Q(x) < \infty]$.

Theorem 1. Suppose that $\mathcal{F} \subset \mathcal{L}_1(Q)$ and $J_{[]}^{(1)}(1) < \infty$. Then as $n \to \infty$

$$\sup_{f \in \mathcal{F}} \Big| G_n^{(i)} f \Big| \xrightarrow{a.s.} 0, \qquad i \in J.$$

Theorem 2. Let class $\mathcal{F}$ such that $\mathcal{F} \subset \mathcal{L}_2(Q)$ and $J_{[]}^{(2)}(1) < \infty$. Then for each $i \in J$ as $n \to \infty$ processes $\{\sqrt{n}\, G_n^{(i)} f, \ f \in \mathcal{F}\}$ converges weakly in $l^{\infty}(\mathcal{F})$ to corresponding mean zero Gaussian process.

These theorems can be used for estimation of unknown parameter $\theta \in \Theta \subseteq \mathbb{R}$. Let $f_\theta : \mathbb{R} \to \mathbb{R}$ be some loss function and $\mathcal{F} = \{f_\theta, \ \theta \in \Theta\}$. For example, (a) in location estimation: $\Theta = \mathbb{R}$ and $f_\theta(x) = (x-\theta)^2$ (estimating the mean); $f_\theta(x) = |x-\theta|^2$ (estimating the median); (b) in maximum likelihood: $\{h_\theta, \ \theta \in \Theta\}$ is a family of densities and $f_\theta(x) = -\log h_\theta(x)$. We estimate θ by M–estimator

$$\theta_n = \mathrm{Arg} \max_{(\theta;i) \in \Theta \times J} \Big\{ \int f_\theta(x) \, dF_{n\tau}(x;i) \Big\}.$$

Assume that θ_n exists. Then under mild conditions on class $\mathcal{F}$, from theorems 1 and 2 one can obtain a strong consistency and asymptotical normality properties of estimator θ_n.

Now consider situation in which $\{f_{h,\theta},\ h \in H,\ \theta \in \Theta\}$ is given collection of measurable functions $f_{h,\theta} : \mathbb{R} \to \mathbb{R}$ indexed by parametrical sets H and Θ. For estimator θ_n we prove that

$$\sup_{h\in H} \left| G_n^{(i)} \left(f_{h,\theta_n} - f_{h,\theta} \right) \right| \xrightarrow{P} 0, \quad i \in J, \quad n \to \infty. \tag{1}$$

The result (1) helps to derive the limit behaviors of estimators $\{F_{n\tau}(\cdot; i)\, f_{h,\theta_n},\, i \in J\}$ by using decomposition

$$\sqrt{n}\left[F_{n\tau}(\cdot; i)\, f_{h,\theta_n} - F_{\tau}(\cdot; i)\, f_{h,\theta}\right] = G_n^{(i)} \left(f_{h,\theta_n} - f_{h,\theta} \right) +$$

$$+ G_n^{(i)}\, f_{h,\theta} + \sqrt{n}\, F_{n\tau}(\cdot; i) \left(f_{h,\theta_n} - f_{h,\theta} \right), \quad i \in J.$$

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Programmed control with probability 1 for stochastic dynamical systems

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A first integral for stochastic equations. Let $(\Omega, F, \{\mathcal{F}_t\}_0^T, \mathbb{P})$ be a probability space with filtration. Suppose that γ is a vector with values in $R_\gamma := \mathbb{R}^{n'}$, $W(t)$ is an $m-$dimensional Wiener process, $\nu(\triangle t; \triangle\gamma)$

is the standard Poisson measure on $[0,T]\times\mathbb{R}^n$ whose values are independent Poissonian random variables on disjoint intervals and sets. The one-dimensional Wiener processes $W_k(t), k=1,\dots,m$, and the Poisson measure $\nu([0;T])$ are defined on the above-mentioned probability space, $\mathcal{F}_t$-measurable, and mutually independent. Note that the random functions appearing below are $\mathcal{F}_t$-measurable and adapt with the above processes.

Let us consider a system of stochastic differential Ito's equations (from now on we use summation convention for repeated indices)

$$\begin{aligned} dX_i(t) = a_i\big(t,X(t)\big)\,dt + b_{i,k}\big(t,X(t)\big)\,dW_k(t) \\ + \int_{R_\gamma} g_i\big(t,X(t),\gamma\big)\nu(dt,d\gamma), \qquad (1) \\ X(0)=X_0, \quad i=\{1,...,n\}, \quad n\geq 2, \end{aligned}$$

under conditions $a_i(t,X)\in\mathcal{C}^{1,1}_{t,x}$, $b_{ij}(t,X)\in\mathcal{C}^{1,2}_{t,x}$, $g_i(t,X,\gamma)\in\mathcal{C}^{1,2,1}_{t,x,\gamma}$, $X=(X_1,\dots,X_n)$.

Refer to a random function $S(t;X;\omega)$ defined on the same probability space as a solution to (1) as a stochastic first integral of the jump diffusion equations system (1) if the following condition holds [1]: $S(t,X(t,X_0,\omega))=S(0,X_0)$ ($\mathbb{P}-$a.s) for all solution $X(t)=X(t,X_0,\omega)$ to system (1).

In case when we consider only *one* realization, a function $s(t;X)=S(t;X;\tilde{\omega})$, $\tilde{\omega}\in\Omega$ is called a first integral of the system (1).

A non-random function $s(t;X)\in\mathcal{C}^{1,2}_{t,x}$ is a first integral of system (1) if and only if it satisfies the conditions [1]:

1. $\dfrac{\partial s(t;X)}{\partial t}+\dfrac{\partial s(t;X)}{\partial X_i}\Big[a_i(t;X)-\dfrac{1}{2}\,b_{j\,k}(t;X)\dfrac{\partial b_{i\,k}(t;X)}{\partial x_j}\Big]=0\ ;$

2. $b_{i\,k}(t;X)\dfrac{\partial s(t;X)}{\partial X_i}=0$, for all $k=\{1,\dots,m\}$;

3. $s(t;X)-s\big(t;X+g(t;\mathbf{x};\gamma)\big)=0$ for all $\gamma\in R_\gamma$.

A generalized Itô–Wentzell formula. This result we obtained using by a Generalized Itô–Wentzell formula or Itô–Wentzell formula with Jumps (2). Let us note: $\partial_{X_i}F=\dfrac{\partial F(t,X)}{\partial X_i}\Big|_{X=X(t,Y)}$ for any function $F(t,X(t,Y))$.

Generalized Itô–Wentzell formula [1,3]: Let $X(t,Y) \in \mathbb{R}^n$ is a solution for SDE (1) and $F\left(t, X(t,\mathbf{y})\right)$ is a stochastic process, $F(t,X) \in \mathcal{C}^{1,2}_{t,x}$. Suppose that a random function $F(t,X,\omega)$ satisfies the equation

$$d_t F(t,X) = Q(t,X)dt + D_k(t,X)dW_k(t) + \int_{R_\gamma} G(t,X,\gamma)\nu(dt,d\gamma)$$

under conditions: $Q(t,X) \in \mathcal{C}^{1,2}_{t,x}$, $D_k(t,X) \in \mathcal{C}^{1,2}_{t,x}$, $G(t,X,\gamma) \in \mathcal{C}^{1,2,1}_{t,x,\gamma}$.

Then it holds:

$$\begin{aligned} d_t F(t,X(t,Y)) &= D_k(t,X(t,Y)) + b_{i\,k}(t,X(t,Y))\,\partial_{X_i}F)\,dW_k(t) \\ &+ (Q(t,X(t,Y)) + a_i(t,X(t,Y))\,\partial_{X_i}F + b_{i\,k}(t,X(t,Y))\,\partial_{X_i}D_k + \\ &+ 2^{-1}\,b_{i\,k}(t,X(t,Y))b_{j\,k}(t,X(t,Y))\partial^2_{X_iX_j}F)\,dt \\ &+ \int_{R_\gamma} G(t,X(t,Y) + g(t,X(t,Y),\gamma))\,\nu(dt,d\gamma) \\ &+ \int_{R_\gamma} [F\big(t,X(t,Y) + g(t,X(t,Y),\gamma)\big) - F(t,X(t,Y))]\,\nu(dt,d\gamma). \quad (2) \end{aligned}$$

Construction of the differential equations system. The conditions for a first integral above allow us to construct a system of stochastic differential Ito's equations (as well as non-stochastic differential equations system) which has a function $s(t,X(t))$ as a first integral [2]. This is proved be the statistical modeling of random processes with invariants [4].

Programmed controls with probability 1 for a dynamical system. Now we can apply our results to control problem for dynamical systems.

A programmed control with probability 1 is called a control of stochastic system which allows to preserve a constant value with probability 1 for the same function which depends on this system's position for any long time periods.

Consider the stochastic non-linear jump diffusion equations system:

$$\begin{aligned} dX(t) = \big(P(t;X(t)) + R(t;X(t))\cdot u(t;X(t))\big)dt \\ + B(t;X(t))\,dW(t) + \int_{R_\gamma} \Xi(t;X(t);\gamma)\,\nu(dt;d\gamma), \quad (3) \end{aligned}$$

where $P(\cdot)$, $R(\cdot)$ are given matrix functions and $B(\cdot)$, $\Xi(\cdot)$ are given or unknown ones. For such systems we construct the programmed control $u(t;X(t))$ with probability 1 (PCP1) which allows the system (3) to

be on the given manifold $\{u(t;X(t))\} = \{u(0;X_0\}$ for each $t \in [0;T]$, $T \le \infty$. The programmed control $u(t;X(t))$ is solution for the algebraic system of linear equations.

Example. Let us construct a PCP1 for a dynamical system

$$dX_1(t) = \big(X_1(t) + X_2(t) + e^{-t} + u_1(t,X(t))\big)\,dt + b_1(t,X(t))dW(t) + \int_{R_\gamma} g_1(t,X(t);\gamma)\,\nu(dt,d\gamma),$$

$$dX_2(t) = \big(X_1(t)X_2(t) + e^{-2t} + u_2(t,X(t))\big)\,dt + b_2(t,X(t))\,dW(t) + \int_{R_\gamma} g_2(t,X(t);\gamma)\,\nu(dt,d\gamma),$$

so a relation $s(t,X(t) = X_2(t)e^{-2X_1(t)} = s(0,X(0)) \equiv s_0$ holds.

Then we obtain:

$$u_1(t,X(t)) = -\frac{f_1(t;X(t))}{f_2(t;X(t)) + 2\,f_3(t,X(t))\,X_2(t)} + 2\,q_{oo}^2(t;X(t))\,e^{-4X_1(t)} - X_1(t) - X_2(t) - e^{-t},$$

$$u_2(t,X(t)) = -\frac{2\,f_1(t;X(t))\,X_2(t)}{f_2(t;X(t)) + 2\,f_3(t;X(t))\,X_2(t)} - X_1(t)\,X_2(t) - e^{-2t},$$

$$b_1(t,X(t)) = q_{oo}(t,X(t))\,e^{-2X_1(t)},$$

$$b_2(t,X(t)) = q_{oo}(t,X(t))\,2\,X_2(t)\,e^{-2X_1(t)},$$

$$g_1(t;X(t);\gamma) = 0.5\,\ln\left[2\,\gamma + e^{2X_1(t)}\right] - X_1(t),$$

$$g_2(t;X(t);\gamma) = 2\,X_2(t)\,\gamma\,e^{-2X_1(t)}.$$

Fig. 1 shows one sample trajectory of the random process $X(t)$ (three coordinates; horizontal line indicates values of the functions $s(t_k,X(t_k))$.

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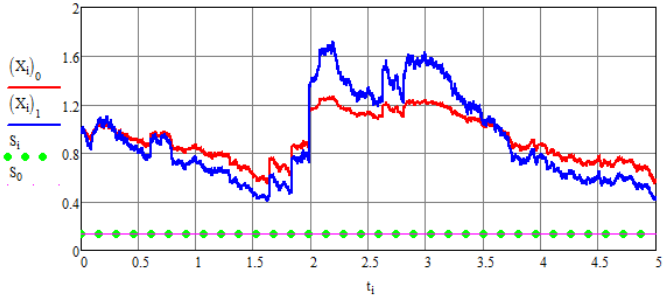

Fig. 1: Sample trajectory of the random process $X(t)$ (coordinates, values of the first integral)

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Quality of service estimation in telecommunication system with nonhomogeneous input flow

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The problem of quality of service estimation is the most important one in telecommunication systems analysis. In our previous work (Khokhlov, Lukashenko, Morozov [2]) using the methodology proposed in the paper of Norros [1] we propose some lower asymptotic estimate of the overflow probability of large buffer when the input is a stream consisting of two independent components: the fractional Brownian motion and stable Levy motion with same Hurst parameters. Now we consider the case of different Hurst parameters.

We consider the single-server fluid queue which is fed by the following input process: $A(t) = mt+\sigma_1 B_{H_1}(t)+\sigma_2 L_\alpha(t),\ t \geq 0$, where where $m > 0$ is the mean input rate; $B_{H_1} = (B_{H_1}(t), t \in R)$ is a fractional Brownian motion (FBM) with Hurst parameter H_1, and $L_\alpha = (L_\alpha(t), t \in R)$ is symmetric α-stable Levy motion. Both processes are self-similar with indexes H_1 and $H_2 = 1/\alpha$ respectively. In what follows we assume that $H_1 \neq H_2$, $1/2 < H_1, H_2 < 1$, $\sigma_1 = \sigma_2 = \sigma$, the processes B_{H_1} and L_α

are independent. We are interested in estimation of so-called *overflow probability*, i.e. the probability that stationary workload Q exceeds some threshold level b, namely $\varepsilon(b) := P[Q > b]$. Denote $H = \min(H_1, H_2)$. Our main result is the following estimate: for large $b > 0$

$$\varepsilon(b) \geq C \cdot b^{-(1-H)\cdot\alpha}.$$

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Statistical procedures for network structures identification with invariant risk function

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Network model of a complex system is a complete weighted graph where nodes corresponds to the elements of the system and weights of edges are given by some measure of connection between them. Network models are widely used in the stock market network analysis Mantegna [1], Boginski [2], Boginski [3]. Nodes of the network model correspons to the stocks of the stock market and weights of edges are given by Pearson correlations between fluctuations of stock returns.

Different network structures which contain a key information of network models are analyzed. Minimum spanning tree (MST) Mantegna [1], planar maximally filtered graph (PMFG) and market graph Boginski [2], Boginski [3] are most popular network structures in market network analysis.

Key problem is to identify these network structures by observations of stocks return fluctuation. Traditional approach to the problem is to

calculate Pearson correlation for any pair of stocks and to apply corresponding algorithms to network structures identification. At the same time the statistical properties of the algorithms such as unbiasedness, invariance, optimality are unknown.

In the presentation the concept of random variables network is introduced. Random variables network is a pair (X, γ) , where vector $X = (X_1, X_2, \ldots, X_p)$ has multivariate distribution and $\gamma = (\gamma_{ij} = \gamma(X_i, X_j))$ is a measure of similarity between X_i and X_j. It is easy to see that traditional approach is based on application of corresponding algorithms of network structures identification to Pearson correlation network.

In the presentation the sign random variables network Kalyagin [4] is introduced which is based on measure $P((X_i-\mu_i)(X_j-\mu_j) > 0)$ - probability of sign coincidence of two random variables X_i and X_j with respect to their shift parameters. It is shown that if vector $X = (X_1, X_2, \ldots, X_p)$ has multivariate elliptically contoured distribution $ECD(\mu, \Lambda, g)$ with known μ then network structures in Pearson correlation network and network structures in sign correlation network are coincide. The procedures for network structures identification in sign correlation network are constructed. It is proved in Kalyagin [4] that these procedures have invariant risk function with respect to function g.

In Koldanov [5] the case of unknown μ is considered. The random variable network with measure $P((X_i(t) - \overline{X}_i)(X_j(t) - \overline{X}_j) > 0)$ is investigated. It is proved that if matrix of observations

$$\begin{pmatrix} X_1(1) & X_1(2) & \ldots & X_1(n) \\ X_2(1) & X_2(2) & \ldots & X_2(n) \\ \ldots & \ldots & \ldots & \ldots \\ X_p(1) & X_p(2) & \ldots & X_p(n) \end{pmatrix}$$

has matrix elliptically contoured distribution Gupta [6] then

$$P((X_i - \mu_i)(X_j - \mu_j) > 0) = P((X_i(t) - \overline{X}_i)(X_j(t) - \overline{X}_j) > 0)$$
$$\forall t = 1, \ldots, n; \quad \forall i, j = 1, \ldots, p, \quad i \neq j.$$

It implies that network structures in network model with measure $P((X_i-\mu_i)(X_j-\mu_j) > 0)$ and network structures in network model with measure $P((X_i-\overline{X}_i)(X_j-\overline{X}_j) > 0)$ are coincide. Moreover it implies the property of invariant risk function of procedures for network structures identification in sign correlation network with respect to unknown μ.

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Probabilistic methods for the analysis of fractional and generalized fractional partial differential equations

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From the point of view of stochastic analysis the Caputo and Riemann–Liouville derivatives of order $\alpha \in (0, 2)$ can be viewed as (regularized) generators of stable Lévy motions interrupted on crossing a boundary. This interpretation naturally suggests fully mixed, two-sided or even multidimensional generalizations of these derivatives, as well as a probabilistic approach to the analysis of the related equations. These extensions are introduced and some well-posedness results are obtained that generalize, simplify and unify lots of known facts. This probabilistic analysis leads one to study a class of Markov processes that can be constructed from any given Markov process in $\mathbf{R}^d$ by blocking (or interrupting) the jumps that attempt to cross certain closed set of 'checkpoints'. As examples we present wide classes of generalized fractional

equations giving probabilistic interpretations of their solutions in terms of the Dynkin type martingales and/or chronological operator-valued extensions of the Feynman–Kac formulas. Main ideas of the talk are discussed in more detail in the publications given below.

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Probability models of statistical regularities in rainfall data

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Mixed probability models are proposed for statistical regularities in the behavior of such characteristics of rainfall data as the duration of

a wet period, maximum daily precipitation within a wet period and total precipitation volume per a wet period. The base for the models is the generalized negative binomial (GNB) distribution. The results of fitting the GNB distribution to real data are presented and demonstrate excellent concordance of the GNB model with the empirical distribution of the duration of wet periods measured in days. Based on this GNB model, asymptotic approximations are proposed for the distributions of the maximum daily precipitation volume within a wet period and of the total precipitation volume for a wet period. The asymptotic distribution of the maximum daily precipitation volume within a wet period turns out to be a tempered scale mixture of the gamma distribution in which the scale factor has the Weibull distribution, whereas the asymptotic approximation for the total precipitation volume for a wet period turns out to be the generalized gamma (GG) distribution. Both approximations appear to be very accurate. These asymptotic approximations are deduced using limit theorems for statistics constructed from samples with random sizes having the generalized negative binomial distribution. Based on these models, two approaches are proposed to the definition of abnormally extremal precipitation. These approaches improve the existing ones [1], [2], [3]. The first approach to the definition (and determination) of abnormally extreme precipitation is based on the distribution of the maximum daily precipitation of the form of a tempered scale mixture of the gamma distribution in which the scale factor has the Weibull distribution. The analytic and asymptotic properties of this distribution are discussed. According to the first approach, a daily precipitation volume is considered to be abnormally extremal, if it exceeds a certain (pre-defined) quantile of this distribution. The second approach is based on that the total precipitation volume for a wet period has the GG distribution. This model is deduced as a version of the law of large numbers for random sums in which the number of summands has the GNB distribution. Hence, the hypothesis that the total precipitation volume during a certain wet period is abnormally large at a given time horizon can be formulated as the homogeneity hypothesis of a sample from the GG distribution. Two equivalent tests are proposed for testing this hypothesis. One of them is based on the beta distribution whereas the second is based on the Snedecor–Fisher distribution. Both of these tests deal with the relative contribution of the total precipitation volume for a wet period to the considered set (sample) of successive wet periods. Within the second approach it is possible to introduce the notions of relatively abnormal and absolutely abnormal precipitation volumes. The

results of the application of this test to real data are presented yielding the conclusion that the intensity of wet periods with abnormally large precipitation volume increases.

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Bayesian method of modeling the balance and advantage processes

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The overwhelming majority of modern life aspects, from household appliances to public administration, have become so complex that the determinnation of performance criteria by deterministic analysis is virtually impossible. Thus, all sorts of indices and ratings are becoming more common allowing to make decisions quickly where a study would take years and require significant financial and material resources. The creation of ratings and indexes is normally based on the separation of the model parameters into two classes. The first class includes parameters that facilitate the functioning of the target object and positively affect the process (*p-factors*); the second class includes parameters that

inhibit and negatively affect the functioning (*n-factors*). Naturally, the functioning of the system under investigation mostly depends not on the explicit values of n- and p-factors, but rather on their ratio. At the same time, a large discrepancy between the values of the factors usually indicates either excessive costs of "fighting negative influence" or underestimated negative impact. Thus, in order to make the system balanced, it is reasonable to strive to the ratio of the n-factor to the p-factor equal to unity. However, the statements of the problem exist where the prevalence of the p-factor over the n-factor is reasonable to achieve despite the costs. For example, in case of security or reliability investigation. In this situations, the ratio of the negative to the positive factor tends to zero, and the ratio of the p-factor to the sum of the p- and n-factors and its closeness to unity should be considered instead in order to understand the closeness to the solution.

Denote by λ and μ respectively the n- and p-factors of the model. Consider the *balance index* $\rho = \lambda/\mu$ and the *advantage index*

$$\pi = \frac{\mu}{\mu + \lambda} = \frac{1}{1 + \rho}.$$

Examples of the balance and advantage indices are found in all kinds of areas of knowledge from demography to simulation of emergencies.

Over the course of time, n- and p-factors, and hence the balance/advantage indices, undergo changes. This is caused by the instability of the environment in which the functioning takes place – economic development, the political system, production technologies, population preferences, etc., – change. For this reason, it makes sense to consider not only the instantaneous values of the factors and indices, but also the corresponding functions of time: the *n-process* $\lambda(t)$, *p-process* $\mu(t)$, *balance process*

$$\rho(t) = \frac{\lambda(t)}{\mu(t)}$$

and *the process of advantage*

$$\pi(t) = \frac{\mu(t)}{\mu(t) + \lambda(t)}.$$

The impossibility of a thorough study of the "states of nature", in which the system under investigation operates, and inevitable errors in measurements are the prerequisites for considering factors, and hence indices, as random variables. Furthermore, one must take into account that

global changes in the environment rarely occur, therefore, the laws that affect the values of the factors can be considered unchangeable within the framework of a particular model. Hence, the distributions of the considered random variables should be assumed to be given a priori.

The above reasoning leads to the application of the Bayesian method to the balance models.

In the report, a number of implementation examples for the balance and advantage indices from specific areas of knowledge are provided. The analytical results for one-dimensional distributions of balance processes for models with a priori gamma-type distributions are presented.

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Numerical results on finite source Markov retrial system with collision

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A retrial queuing system with a single server is investigated in this paper. The server is subject to random breakdowns. The results are provided by the help of recursive numerical calculations (Bérczes and Sztrik and Tóth and Nazarov [1], Kim [2], Wang and Zhao and Zhang [5]). The number of sources of calls is finite and collision (Nazarov and Kvach and Yampolsky [3], Tóth and Bérczes and Sztrik and Kvach [4]) can take place. The failure of server block the system's operation therefore the arriving customers can not enter the system, meaning that those calls are lost. All the random variables included in the model construction are assumed to be generally distributed and independent of each other. From the Kolmogorov system equations a recursive algorithm has been derived for non-blocking case ([3]). As the novelty of this analysis, this algorithm is modified to the blocking case, as well. Various figures represent the impact of blocking phenomenon on the main performance measures like mean and variance of number of customers in the system, mean and variance of response time, mean and variance of time a customer spent in service, mean and variance of sojourn time in the orbit.

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Clustering and assortativity in configuration graphs

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Since the end of the XX-th century the study of random graphs with node degrees being independent identically distributed random variables following a common power-law distribution has gained steam. The reason was quite obvious: observations of real-world complex communication networks showed (see e.g. Faloutsos etc. [1], Hofstad [2]) that these

models could be used for their description. However, with networks' growth it has become obvious that it is not enough to know the node degree distribution and its parameters to get a good-fit model of a real network, but there are some numerical characteristics that have to fit in also.

In this work we consider configuration graphs introduced by Bollobas [3] with the following power-law node degree distribution (see Reittu and Norros [4])

$$\mathbf{P}\{\xi = k\} = k^{-\tau} - (k+1)^{-\tau}, \qquad \tau > 1,\ k = 1, 2, \ldots,$$

where ξ is a random variable equal to the degree of an arbitrary node. Node degrees form incident semiedges numbered in an arbitrary order and the graph is constructed by an equiprobable joining of all semiedges one to another to form links. Obviously, such construction supposes the sum of node degrees to be even, so if otherwise one semiedge is added to an equiprobably chosen node to form a lacking connection. Configuration model allows loops and multiple links in its graph.

Recent works (see e.g. Biaconi and Barabasi [5], Pavlov [6]) that the node degree distribution can not only change with the growth of a network size but even be random, which means that the graph is constructed in a so called random environment. Thus, in our work we consider two types of configuration graphs. The first one with the parameter τ of the distribution (1) being a fixed value and the second one with the values of τ being determined separately for each node from either uniform or truncated normal distribution on some predefined interval (a, b), $1 < a < b < \infty$, so we can say that the graph is formed in random environment.

Along with the node degree distribution description of real-world complex networks includes studying various numerical characteristics that show both local and global network properties. The best known among them are global and local clustering coefficients and assortativity coefficient.

Assortativity coefficient A is used for estimating correlation between the degrees of incident nodes, wherefore it is proposed (see e.g. Newman [7]) to use Pearson correlation coefficient for this purpose. Obviously, if nodes with high degrees connect mostly to nodes with also high degrees, then the assortativity coefficient A will be positive and the network is called assortative, otherwise the coefficient will be negative and a corresponding network is called disassortative.

For estimating the degree of graph clusterization we used the following global C_G and network average C_L clustering coefficients (see Newman [7]):

$$C_G = \frac{3 \times \text{number of graph triangles}}{\text{number of connected triples of nodes}},$$

$$C_L = \frac{1}{N}\sum_{i=1}^{N} C_i,$$

where

$$C_i = \frac{\text{number of triangles connected to node i}}{\text{number of triples centered on node i}},$$

where a "triple" means a single node connected by links to two others, C_i is local clustering coefficient (Newman [7]). Since configuration graphs may have loops and multiple links, in calculating clustering coefficients loops are not counted and multiple links are considered as one.

The results were obtained by simulation technique. We considered configuration graphs with the number of nodes $100 < N < 10000$ in two cases of the node degree distribution: with fixed values of $1.01 \leq \tau \leq 2.5$ and random environment, when τ was either uniformly distributed on a predefined interval $[a, b]$ or was a random variable following a truncated normal distribution on the same interval (a, b) with the expectation of ξ at each interval (a, b) being defined as the middle value $(a + b)/2$ and the standard deviation $\sigma = (b-a)/6$ in accordance with the three-sigma rule. The considered intervals (a, b) were the following: $(1, 2)$, which corresponds to a well-known property of communication networks (Hofstad [2]), $(2, 3)$, connected with forest fire modeling (Leri and Pavlov[8]) and $(1, 3)$ as a generalization of the first two. Based on the obtained results we derived regression dependencies of coefficients A, C_G and C_L on the graph size N and the parameter of the node degree distribution τ in the first considered case, when τ was fixed. The general form of the obtained equations looked like the following (here and in what follows CF denotes either of the three considered coefficients):

$$CF = c \cdot N^{-d+h/\tau},$$

where the coefficient c was negative in the relation for assortativity coefficient A, which means that configuration graphs are to be used for modeling only disassortative networks, and for clustering coefficients C_G and C_L c was positive. The coefficients d and h were always positive. Determination coefficients for all models were greater than 0.95.

In the case of random environment we also obtained regression relations of the coefficients A, C_G and C_L on the graph size N. The general form of these equations was derived to be as follows:

$$CF = p \cdot N^{-q},$$

where the coefficient p was negative in the relation for the coefficient A and positive for C_G and C_L. Coefficient q was positive in all cases and $R^2 \geq 0.97$ for all models.

We believe that these results will be helpful in constructing models of specific networks in the form of configuration graphs with the power-law node degree distribution (1) by choosing the best fitting values of the parameter τ or by choosing the distribution of a random τ fitting the real values of the assortativity and clustering coefficients of these networks. Moreover, we compared the values of A, C_G and C_L calculated for real-world networks and given by Newman [7] with the same coefficients for the corresponding configuration graphs of the same size obtained from our equations. The results showed that for modeling of the Internet on AS-level configuration graphs with $1.02 \leq \tau \leq 1.17$ give the best fit, while for modeling of some social networks the value of τ must be greater than 2.

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On generalization of the Ahmad–Wang inequality

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Let X_1, X_2, …, X_n be independent random variables on a common probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with $\mathbb{E}X_k = 0$, $\mathbb{E}X_k^2 = \sigma_k^2$, and $B_n^2 = \sum_{k=1}^n \sigma_k^2 > 0$. Denote

$$\mu_k(z) = \mathbb{E}X_k^3 \mathbf{1}(|X_k| < z), \quad \lambda_k(z) = z \cdot \mathbb{E}X_k^2 \mathbf{1}(|X_k| \geqslant z), \quad z > 0,$$

$$F_n(x) = \mathbb{P}(X_1 + X_2 + \ldots + X_n < xB_n), \quad \Delta_n = \sup_{x \in \mathbb{R}} |F_n(x) - \Phi(x)|,$$

where Φ stands for the standard normal distribution function. Let $\mathcal{G}$ denote the set of all nondecreasing functions $g : \mathbb{R}_+ \to \mathbb{R}$ such that $x/g(x)$ is nondecreasing for $x > 0$. We prove that for every $\varepsilon \in (0; +\infty]$, $\gamma > 0$ there exist constants $C(\varepsilon, \gamma)$, $\widetilde{C}(\varepsilon, \gamma)$ depending only on ε, γ such that

$$\Delta_n \leq \frac{C_1(\varepsilon, \gamma)}{B_n^2 g(B_n)} \cdot L_{n,1}(g, \varepsilon, \gamma), \quad \Delta_n \leq \frac{C_2(\varepsilon, \gamma)}{B_n^2 g(B_n)} \cdot L_{n,2}(g, \varepsilon, \gamma),$$
$$\forall g \in \mathcal{G},$$

where

$$L_{n,1}(g, \varepsilon, \gamma) = \sup_{0 < z < \varepsilon B_n} \frac{g(z)}{z} \left(\left| \sum_{k=1}^n \mu_k(z) \right| + \sum_{k=1}^n \lambda_k(z) \right),$$

$$L_{n,2}(g, \varepsilon, \gamma) = \frac{g(\varepsilon B_n)}{\varepsilon B_n} \left| \sum_{k=1}^n \mu_k(\varepsilon B_n) \right| + \sup_{0 < z < \varepsilon B_n} \frac{g(z)}{z} \sum_{k=1}^n \lambda_k(z).$$

The obtained inequalities improve and generalize those in [1–4] and, according to Zolotarev's [5] classification, can be called *natural* convergence rate estimates in the Lindeberg–Feller theorem.

Let X_1, X_2, ..., X_n have the same distribution function $F(x)$. For $i = 1, 2$ denote asymptotically exact constants, which are lower bounds for $C_i(\varepsilon, \gamma)$:

$$C_{AB,i}(g, \varepsilon, \gamma) = \sup_F \limsup_{n\to\infty} \frac{\Delta_n(F)}{L_{n,i}(F, g, \varepsilon, \gamma)},$$

$$\overline{C}_{AE,i}(g, \varepsilon, \gamma) = \limsup_{n\to\infty} \sup_F \frac{\Delta_n(F)}{L_{n,i}(F, g, \varepsilon, \gamma)},$$

$$C_{AE,i}(g, \varepsilon, \gamma) = \limsup_{\ell\to 0} \sup_{n, F: L_{n,i}(F,g,\varepsilon,\gamma)=\ell} \frac{\Delta_n(F)}{\ell},$$

$$\underline{C}_{AE,i}(g, \varepsilon, \gamma) = \limsup_{\ell\to 0} \limsup_{n\to\infty} \sup_{F: L_{n,i}(F,g,\varepsilon,\gamma)=\ell} \frac{\Delta_n(F)}{\ell},$$

$$C^*_{AE,i}(g, \varepsilon, \gamma) = \sup_{\ell>0} \limsup_{n\to\infty} \sup_{F: L_{n,i}(F,g,\varepsilon,\gamma)=\ell} \frac{\Delta_n(F)}{\ell},$$

where $g \in \mathcal{G}$, $\varepsilon \le 1$, $\gamma > 0$, $i = 1, 2$. We provide lower bounds for each of the above asymptotically exact constants.

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A new class of spatial survival model with closed skewed-Gaussian random effect

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Abstract. Random effects of frailty components are used in survival models to enter the unknown risk factors. But in many cases, there may be a spatial correlation between the survival times. In this case, a Gaussian random field is usually considered for random effects while entering this component to the model convert survival model to spatial survival model. But the consideration of a Gaussian random field for spatial random effects sometimes not correspond to reality. In this paper, by considering a closed skew Gaussian random field for random effects we propose a new class of spatial survival models. In a simulation study, we will show that the deviation from the Gaussian assumption random effects have an undesirable effect on parameters estimation in the spatial survival model, while the use of the closed skew Gaussian random effects provides more accurate parameters estimates. Finally, the introduced model is applied to explore the pattern of infecting Cercosporiose in olive trees.

Keyword Frailty, Spatial Survival Data, Closed Skew Gaussian Random Field, Cercosporiose.

Introduction. Survival analysis has a long history in medical studies and reliability in engineering Cox and Oakes [1]. It is usually assumed in survival models that the failure times of the subjects are independent. while in many cases this assumption is not realistic in some applications and the failure times are spatially correlated . Many Scientific researcher Biggeri et al [2] and Ramsay et al [3] have shown that in the presence of spatial correlation in survival data and ignoring it in modeling and analyzing survival data can lead to false and misleading results. Random effects are usually a latent component of the survival data, that can be achieved by recognizing the spatial correlation and considering through a spatial survival model to yield results consistent with reality. The analysis of survival models with spatial random effects has a history of

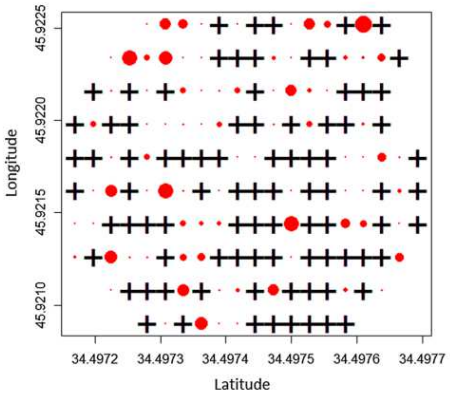

Fig. 1: The locations of trees in the garden and time disease, circle areas are proportional to observations and pluses represent censored data

less than two decades. Most spatial survival models are introduced by researchers are suitable for lattice data that the spatial correlation exists between the areas containing survival data. But in geostatistical cases, the analysis of survival models are complicated in parameter estimation. Motarjem et al [4] introduced a spatial survival model for analysing geostatistical survival data, where a Gaussian random field is used for considering the spatial random effects. However, due to the existence of skewness in survival data, the Gaussian assumption of random effects may not be realistic. In this paper, by considering a closed skew Gaussian random field for spatial random effect, a new model for skew spatial random effects is proposed, the effect of this deviation on the estimation of model parameters is investigated.

Application. In this study, Cercorpiose disease infestation in a garden with an area of 5000 m^2 in which 173 olive planets exist, is studied daily for two months. Age (in years), type (two type) and height (in meter) of each olive tree considered as covariates. In the case of having tree disease, the disease time noted. By the end of the study, 85 trees have been infected and the others were right censored. Consequently, we have 51 percent right censoring. The location of trees showed in Fig. 1 while the infected trees demonstrated by circles and the others by plus signs (+). The area of each circle relates to infecting time in a way that smaller circles indicate earlier infection and larger circles depict later infection.

Table 1: Parameter estimates of Cox and Frailty models fitted to olivegrowning data

	Proportional hazards		Frailty	
Par.	Est.	SE	Est.	SE
β_1	-0.381	0.129	-0.600	0.192
β_2	-0.662	0.156	-1.026	0.221
β_3	0.351	0.119	0.477	0.181
$ln(\hat{L})$	-396.667		-340.046	
AIC	799.334		682.092	

Table 2: Parameter estimates of proposed model with different covariance functions

	Exponential		Gaussian		Spherical	
Par.	Est.	SE	Est.	SE	Est.	SE
β_1	-0.592	0.014	-0.573	0.017	-0.503	0.021
β_2	-1.402	0.059	-1.203	0.075	-1.011	0.089
β_3	0.624	0.073	0.602	0.082	0.589	0.102
a	0.999	0.083	0.973	0.098	1.121	0.108
σ^2	0.307	0.039	0.296	0.052	0.213	0.083
δ	0.512	0.084	0.419	0.102	0.408	0.106
$ln(\hat{L})$	-293.084		-301.209		-304.284	
AIC	598.168		614.418		620.568	

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Asymptotic representation for likelihood ratio statistics in competing risks model under hybrid censoring

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The usefulness of concept of local asymptotic normality (LAN) of a family
of probability distributions in problems of theory of asymptotic estimation and hypothesis testing has been demonstrated in a number of papers. LAN is a property of a sequence of statistical models, which allows this sequence to be asymptotically approximated by a normal location model, after a rescaling of the parameter. The notation of LAN was introduced by Le Cam [1] in the case of independent and identically distributed sampling from a regular parametric model. Several extensions of property of LAN for dependent and nonidentically distributed sampling schemes has been established in statistical literature. In the papers [2-4] the concept of LAN extended in the competing risks model (CRM) under random censoring of observations from the right, both sides and by nonobservation intervals. In this paper we discuss property of LAN in the CRM by progressively hybrid censored data.

In the CRM our interest is focused on random variable (r.v.) X with values from measurable space $(\mathcal{X}, \mathcal{B})$ and pairwise disjoint events $\left(A^{(1)}, ..., A^{(k)}\right)$, where for a fixed k, $P(\bigcup\limits_{i=1}^{k} A^{(i)}) = 1$. In survival analysis X means survival time of object (individual, physical system) exposed to k competing risks and failing in case one of the events $A^{(i)}$, $i = \overline{1,k}$. The pairs $(X, A^{(i)}), i = \overline{1,k}$, denote the time and reason the object fails. Let $\left\{(X_j, A_j^{(1)}, ..., A_j^{(k)}),\ j \geq 1\right\}$ be independent copies of ensemble $(X; A^{(1)}, ..., A^{(k)})$ during the experiments under homogenous conditions. Let $\delta_j^{(i)} = I(A_j^{(i)})$ is a indicator of event $A_j^{(i)}$. Every vector $\zeta_j = (X_j, \delta_j^{(1)}, ..., \delta_j^{(k)})$ induces a statistical model with sample the space $\mathcal{Y} = \mathcal{X} \times \{0,1\}^{(k)}$ and σ - algebra $\mathcal{C}$ of sets of the from $B \times D_1 \times ... \times D_k$, where $B \in \mathcal{B}$ and $D_i \subset \{0,1\}$, $i = \overline{1,k}$. Suppose that distribution of the vector ζ_j on $(\mathcal{Y}, \mathcal{C})$ depends on an parameter $\theta = (\theta_1, ..., \theta_s) \in \Theta$: $Q_\theta(x, y^{(1)}, ..., y^{(k)}) =$

$P_\theta(X < x, \delta^{(1)} = y^{(1)}, ..., \delta^{(k)} = y^{(k)})$, $x \in R^1, y^{(i)} \in \{0,1\}$. Let $H(x;\theta) = P_\theta(X < x)$, $H^{(i)}(x;\theta) = P_\theta(X < x, \delta^{(i)} = 1)$, $i = \overline{1,k}$. Obvious that $H^{(1)}(x;\theta) + ... + H^{(k)}(x;\theta) = H(x;\theta)$.Let $H^{(i)}(x;\theta)$ are absolutely continuous, $h^{(i)}(x;\theta) = \frac{\partial H^{(i)}(x;\theta)}{\partial x}$, $i = \overline{1,k}$ and $h(x;\theta) = h^{(1)}(x;\theta) + ... + h^{(k)}(x;\theta)$. Let $X_{1n} < X_{2n} < ... < X_{nn}$ are order statistics of subsample $\{X_1, ..., X_n\}$ and $\{\delta_{jn}^{(1)}, ..., \delta_{jn}^{(k)}, j = \overline{1,n}\}$ indicator functions in sample $\zeta^{(n)} = (\zeta_1, \ldots, \zeta_n)$ corresponding to order statistics $\{X_{jn}, j = \overline{1,n}\}$. Denote , where $\zeta_{jn} = (X_{jn}, \delta_{jn}^{(1)}, ..., \delta_{jn}^{(k)})$. Throughout $(\mathcal{Y}^{(n)}, \mathcal{U}^{(n)}, Q_\theta^{(n)})$ denote the sequence of statistical experiments, induced by $(Z^{(1)}, ..., Z^{(n)})$. Now we consider the experimental situation when the competing risks data is random hybrid censored from the right. A hybrid censoring, which is the mixture of type I and II censoring, can be used to save resources. If the experiment stops either at a fixed time $T \in R$, the experiment is called type I (or (n,T)) censoring model or is continues until $r(0 < r < n)$ failures occur, the experiment is called type II (or (n,r)) censoring model. If the experiment is continues until either r failures occur or test duration T is reached, whichever comes first, we call the experiment a hybrid (or (n,r,T)) censoring model. In considered competing risks situation the hybrid censored CRM we denote as $(n,r,T)^k$. In $(n,r,T)^k$-model competing risks data is censored from the right by r.v. $T_{rn} = \min\{T, X_{rn}\}$. In $(n,r,T)^k$-model a number τ of observed data is r.v.:

$$\tau = \begin{cases} \sup\{m : X_{mn} \le T,\ m \le r\}, & \text{if } X_{1n} \le T, \\ 0, & \text{otherwise.} \end{cases}$$

Thus the observed data $\left(Z^{(\tau)}, \tau\right)$ have a joint density function

$$p_n\left(Z^{(\tau)}, \theta\right) = \frac{n!}{(n-\tau)!} \prod_{l=1}^{\tau} \prod_{i=1}^{k} \left\{ \left[h^{(i)}(x_{in};\theta)\right]^{y_{ln}^{(i)}} \right\} [1 - H(t_{rn};\theta)]^{n-\tau} \times$$

$$\times I\left(x_{1n} < ... < x_{rn},\ \tau \le r\right),$$

where $t_{rn} = \min\left\{x_{rn}, T\right\}$.

Let

$$l_n^{(\tau)}(u) = \frac{p_n(Z^{(\tau)};\theta_n)}{p_n(Z^{(\tau)};\theta)}$$

is a likelihood ratio statistics, where $\theta_n = \theta + un^{-1/2} \in \Theta$ and θ, u held fixed. Under certain regularity conditions on underlying distribution family and on stopping time τ the LAN property of experiment is

established:

$$l_n^{(\tau)}(u) = \exp\Big\{u\,(I(\theta))^{1/2}\,\Delta_n^{(\tau)} - \frac{u^2}{2}\,I(\theta) + R_n(u)\Big\},$$

where $R_n(u) \overset{Q_\theta}{\to} 0$, $\mathcal{L}(\Delta_n^{(\tau)}/Q_\theta) \to \mathcal{L}(\xi)$, $n \to \infty$, $\xi \overset{D}{=} N(0,1)$ and $I(\theta)$ is Fisher information.

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Some probabilistic features of the iterated Brownian motion

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We give a review of the basic facts about the iterated Brownian motion ${}_{\mu_2}^{\mu_1}I(t) = B_1^{\mu_1}(|B_2^{\mu_2}(t)|)$ where $B_j^{\mu_j}, j = 1, 2$ are two independent Brownian motions with drift μ_j. We study the last zero crossing of ${}_{\mu_2}^{\mu_1}I(t)$ and for this purpose we derive the last zero-crossing distribution of the drifted Brownian motion.

We derive also the joint distribution of the last zero crossing before t and of the first passage time through the zero level of a Brownian motion with drift μ after t. All these results permit us to derive explicit

formulas for ${}^{I}_{\mu}T_0 = \sup\{s < \max_{0\le z\le t}|B_2(z)| : B_1^{\mu}(s) = 0\}$. Also the iterated zero-crossing ${}^{\mu_1}T_{0,{}^{\mu_2}T_{0,t}}$ is analyzed and extended to the case where the level of nesting is arbitrary.

The iterated Brownian motion has been examined from many view points including its connection with fractional equations and some probabilistic properties as the iterated logarithm law.

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New scheme for calculation of senior moment functions for the state vector of linear stochastic delay differential system excited by additive and multiplicative white noises

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Current theoretical studies aimed at studying effects of time-lags on the state and behavior of various systems, began in the middle of the twentieth century [1, 2]. But they began to develop especially intensively only recently that is related to practical needs. Among the first applications of such studies were methods of solving problems of control, and then tasks of biology, mechanics, physics, chemistry, medicine, economics, atomic energy, information theory etc.

Mathematical models for describing phenomena in these areas are constructed in the form of functional-differential equations (FDEs) (see [3, 4] et al.) and various special forms of FDEs such as retarded ordinary and partial differential equations [5–7] including delay and neutral differential equations (DDEs, NDEs) as well as integro-differential equations (IDEs) [8–11].

At present, a considerable interest is being paid to stochastic FDEs (SFDEs) of different types [12–15]. As it happened earlier for deterministic systems, the development of research methods for such equations became important for theory and practice. Analysis of SFDEs causes significant difficulties, since these SFDEs that arise in many applications,

can not be solved exactly. Therefore, the actual task is the development of effective both direct, i.e., obtaining realizations of strong solutions, and indirect, i.e., computation of statistical characteristics, approximate analytical and numerical algorithms for analyzing systems of SFDEs.

Now there is a rather wide class of methods for solving deterministic FDEs [16, 17]. Approximate algorithms of direct numerical integration of SFDEs. of various types (for example [18–20]) are based on these schemes and special compound methods for numerical solution of stochastic ordinary differential equations (SODEs) [21–23].

But there are some forms of SODEs that don't require a very complicated scheme to be examined. One of such the forms is a system of linear SODEs excited by additive and multiplicative white noises. In this case deterministic ODEs for the first and senior (central) moment functions can be obtained exactly in the closed form, i.e., an ODE for a moment function of a current order does not contain moment functions of higher orders. If we now turn to linear stochastic ordinary DDEs (SODDEs) with the same input fluctuations, then we formally will be in a similar situation with respect to a closure of the equations for the moment functions as above. The difference is in the fact that these equations will be ODDEs. To obtain ODEs for senior moment functions without delays, we apply a modification of our scheme [24–26] combining the classical method of steps and extension of the system state space (MSESP).

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Integral representation of the density of the fractional-stable law

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The characteristic function of the fractional stable law has the form

$$\tilde{q}(t,\alpha,\beta,\theta) = E_\beta(\psi(t,\alpha,\theta)), \tag{1}$$

where $E_\beta(z)$ is the Mittag–Leffler function

$$E_\beta(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(1+\beta n)}, \quad \beta > 0, \quad z \in \mathbb{C},$$

$\psi(t,\alpha,\theta) = -|t|^\alpha \exp\{-i(\pi\alpha\theta/2)\operatorname{sign} t\}$, and the parameters are varying within the limits

$$0 < \alpha \leqslant 2, \quad 0 < \beta \leqslant 1, \quad |\theta| \leqslant \min(1, 2/\alpha - 1).$$

The inverse Fourier transformation of the characteristic function (1) was carried out for obtaining of the integral representation of the density of the fractional stable law

$$q(x,\alpha,\beta,\theta) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{-itx}\, \tilde{q}(t,\alpha,\beta,\theta)\, dt.$$

As a result for the density of the fractional stable law the following theorem is valid

Theorem 1. *For any allowed value of the parameters (α,β,θ), such as $0 < \alpha/\beta \leqslant 2$, $|\theta| \leqslant \min(1, 2\beta/\alpha - 1)$, the density of the fractional stable law $q(x,\alpha,\beta,\theta)$ has the form*

$$q(x,\alpha,\beta,\theta) = \frac{\sin(\pi\beta)}{\pi\beta} \int\limits_0^{\infty} \frac{y^{-1/\alpha} g(x\, y^{-1/\alpha}, \alpha/\beta, \theta)}{y^2 + 2\, y\, \cos(\pi\beta) + 1}\, dy, \tag{2}$$

$$\beta \neq 1,\ x \neq 0,$$

$$q(x,\alpha,1,\theta) = g(x,\alpha,\theta), \quad \beta = 1,$$

where $g(x,\nu,\theta)$ *is the density of the strictly stable law* (*see* Zolotarev [1]),

$$g(x,\nu,\theta)=\frac{\nu}{\pi|\nu-1|}\int\limits_{-\pi\theta^*/2}^{\pi/2}\exp\left\{-|x|^{\frac{\nu}{\nu-1}}U(\psi,\nu,\theta^*)\right\}U(\psi,\nu,\theta^*)|x|^{\frac{1}{\nu-1}}\,d\psi,$$

$$\nu\neq 1,$$

$$g(x,1,\theta)=\frac{\cos(\pi\theta/2)}{\pi(x^2-2x\sin(\pi\theta/2)+1)},\qquad g(x,1,\pm1)=\delta(x\pm1).$$

Here $\theta^*=\theta\operatorname{sign}x$,

$$U(\psi,\nu,\theta)=\left(\frac{\sin\left(\nu\left(\psi+\frac{\pi}{2}\theta\right)\right)}{\cos\psi}\right)^{\frac{\nu}{1-\nu}}\frac{\cos\left(\psi(\nu-1)+\frac{\pi}{2}\nu\theta\right)}{\cos\psi}.$$

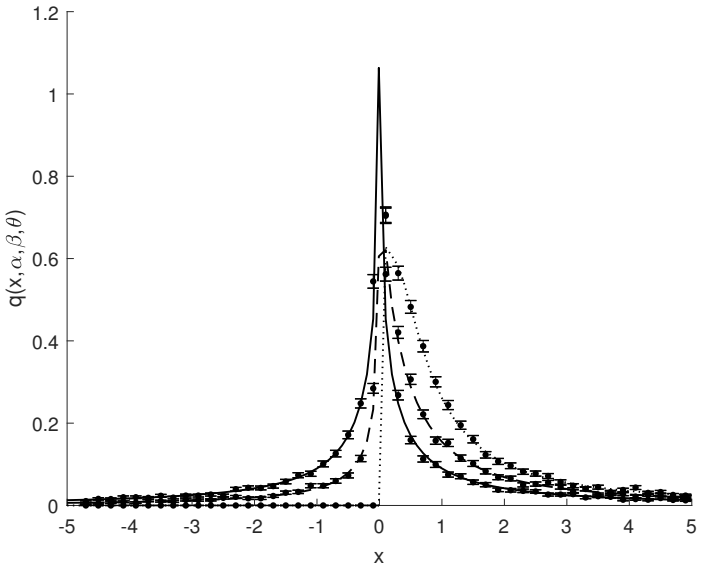

Fig. 1: The densities of the fractional stable laws for the values of the parameters $\alpha=0.6$, $\beta=0.8$, $\theta=1$ (dotted curve), $\theta=0.5$ (dashed line), $\theta=0$ (solid curve)

The results of calculation of the densities are shown on the Fig. 1. The curves are densities obtained by Eq. (2) and the dots are results

obtained by Monte Carlo method. For calculation of the density by Monte Carlo method the following formula is used

$$X(\alpha,\beta,\theta)=\frac{Y(\alpha,\theta)}{(S(\beta,1))^{\beta/\alpha}},$$

where random variables $Y(\alpha,\theta)$ and $S(\beta,1)$ are distributed according to laws $g(x,\alpha,\theta)$ and $g(y,\beta,1)$ respectively.

We can obtain expression for the cumulative distribution function $Q(x,\alpha,\beta,\theta)$ using the Theorem 1. As a result the cumulative distribution function has the form

$$Q(x,\alpha,\beta,\theta)=\frac{\sin(\pi\beta)}{\pi\beta}\int\limits_0^\infty \frac{G(xy^{-1/\alpha},\alpha/\beta,\theta)}{y^2+2y\cos(\pi\beta)+1}\,dy,\quad x>0,$$

$0<\alpha/\beta\leqslant 2$, $|\theta|\leqslant\min(1,2\beta/\alpha-1)$ and $G(x,\nu,\theta)$ is cumulative distribution function of the strictly stable law (Zolotarev [1])

$$G(x,\nu,\theta)=1-\frac{1-\theta^*}{4}(1+\operatorname{sign}(1-\nu))$$
$$+\frac{\operatorname{sign}(1-\nu)}{\pi}\int\limits_{-\pi\theta^*/2}^{\pi/2}\exp\left\{-|x|^{\frac{\nu}{\nu-1}}U(\psi,\nu,\theta^*)\right\}d\psi,\quad \nu\neq 1$$
$$G(x,1,\theta)=\frac{1}{2}+\frac{1}{\pi}\arctan\left(\frac{x-\sin(\pi\theta/2)}{\cos(\pi\theta/2)}\right).$$

The case $x<0$ can be obtained from the relation

$$Q(-x,\alpha,\beta,\theta)=1-Q(x,\alpha,\beta,-\theta).$$

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Thresholding rules in the models with non-Gaussian noise

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Many modern methods of analyzing and processing signals and images are based on the possibility to economically represent the function of a useful signal in a certain basis. For a fairly wide class of functions, this possibility is achieved with the help of wavelet bases, which ensure adaptation to functions that have different degrees of regularity in different regions. This makes it possible to efficiently separate the noise from the useful signal and to remove it using simple thresholding procedures, that is, zeroing out a part of the wavelet coefficients, which are assumed to contain mostly noise. The classical model of observations assumes the presence of white Gaussian noise. In this case, the properties of the estimates obtained by threshold processing are well studied, and the order of the mean-square risk for various classes of functions is calculated [1]. Some results have also been obtained that describe the asymptotic behavior of the mean-square risk estimate, constructed from noisy observations [2].

This report considers a wider class of possible noise distributions, in particular, distributions having heavier tails than Gaussian distribution. For this class, the values of the universal threshold in the methods of hard and soft thresholding are calculated, its asymptotic properties are studied and it is shown that the order of the mean-square risk is close to the minimum up to the logarithm of the number of observations in a power depending on the distribution parameters [3]. Also within the framework of the model under consideration, the strong consistency and asymptotic normality of the mean-square risk estimate for the universal threshold processing are proved under the assumption that the signal function belongs to the Lipschitz class.

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Robust minimax estimation of location and least favorable distributions under asymmetry

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This talk is partially a review of basic former results on the application of Huber's minimax approach to robust estimation of location with the corresponding least favorable (informative) distributions both in the univariate and multivariate cases [1-3], and partially it is a presentation of several recent results and novel problem settings on these issues with a certain accent on the asymmetry of distribution models.

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Integral transforms of characteristic functions and their properties

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We propose a natural generalization of the zero bias transformation, the term introduced by [2], of a probability distribution with non-zero mean in terms of characteristic functions coming back to ideas of Lukacs in the second edition of his celebrated monograph [4]. We discuss this generalization with the other ones called 'non-zero biased' and 'generalized zero biased' [1]. We also introduce other integral transformations of probability distributions, in particular, generalizing the *stationary renewal* distribution (*equilibrium*, or *integrated tail* distribution) and

symmetric equilibrium distribution and study their properties including construction of optimal estimates for the minimal L_1-bounds between the original distribution and its transformation. As corollary and using results of [5] we prove new and sharp moment-type estimates for characteristic functions and their derivatives improving, in particular, some results of [6] and [7].

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Non-arcsine law for random walk conditioned to reach a high level

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Let X_i be i.i.d. random variables with $\mathbf{E}X_i = 0$, $\mathbf{D}X_i < \infty$. Consider the random walk $S_n = X_1 + ... + X_n$. Let $M_n = \max(S_i, i \leq n)$ be its

maximum, let $\tau_M = \min\{i : S_i = M_n\}$ be the first moment the random walks reaches its maximum.

Well-known arcsine law for τ_M states that

$$\mathbf{P}\left(\frac{\tau_M}{n} \le x\right) \to \frac{2}{\pi} \arcsin\sqrt{x},\ x \in [0,1],\ n \to \infty,$$

or in local form that

$$\mathbf{P}\left(\tau_M = m\right) \sim \frac{1}{\pi n \sqrt{\frac{m}{n}\left(1 - \frac{m}{n}\right)}},\ n \to \infty,$$

uniformly by $m/n \in [\varepsilon, 1-\varepsilon]$ for any $\varepsilon > 0$.

Let introduce several classes of random variables.
A random variable X is called *arithmetic* if $\mathbf{P}(X \in \alpha\mathbb{Z}) = 1$ for some $\alpha > 0$. The maximal possible α is called *the arithmetic step* of X.
A random variable X is called *lattice* if $\mathbf{P}(X \in \beta + \alpha\mathbb{Z}) = 1$ for some $\alpha > 0$, β. The maximal possible α is called *the lattice step* of X.
A random variable X is called *strongly arithmetic* if X is arithmetic and its arithmetic step is equal to its lattice step.

For example, if $\mathbf{P}(X = 1) = \mathbf{P}(X = -1) = 1/2$, then X is arithmetic, its arithmetic step is equal to 1, its lattice step is equal to 2, therefore it's not a strongly arithmetic random variable.

In the sequel we consider strongly arithmetic or non-lattice random variables X. Without loss of generality, further on below we suppose that the arithmetic step of X is equal to 1.

Consider the following generalization of the previous problem: to find the asymptotics of probabilities $\mathbf{P}(\tau_M = m |\, M_n = k)$, $\mathbf{P}(\tau_M \ge x |\, M_n = k)$ as $m, k, n \in \mathbb{N}$, n, k, m, $x \to \infty$ in strongly arithmetic case and

$$\mathbf{P}(\tau_M = m |\, M_n \in [y, y + \Delta_n)),\ \mathbf{P}(\tau_M \ge x |\, M_n \in [y, y + \Delta_n))$$

as $n, m \in \mathbb{Z}$, $y \in \mathbb{R}^+$, $n, m, y, x \to \infty$ in non-lattice case, where Δ_n is some sequence, tending to 0 as $n \to \infty$.

In non-lattice case we use integro-local form of limit theorems, introduced by Stone in [1].

We consider three cases: standard deviations ($k, y \in [a\sqrt{n}, b\sqrt{n}]$ for some $0 < a < b$), large deviations ($k, y \in [an, bn]$ for some $0 < a < b$) and moderate deviations ($k, y \in [a_n, b_n]$, $a_n/n^{1/2+\delta} \to \infty$, $b_n/n^{1-\delta} \to 0$ for some $\delta > 0$). For simplicity, we state Theorems 1-5 only for strongly arithmetic case, in non-lattice case the results are similar.

1) Standard deviations.

Theorem 1. Let X_i be strongly arithmetic random variables with $\mathbf{E}X_i = 0$, $\mathbf{D}X_i < \infty$. Then

$$\mathbf{P}(\tau_M = l \,|\, M_n = k) \sim \frac{1}{n} g_1\left(\frac{k}{\sigma\sqrt{n}}, \frac{l}{n}\right), \; n \to \infty,$$

uniformly by $k, l \in \mathbb{Z}$, $l/n \in [a, b]$, $k \in [c\sqrt{n}, d\sqrt{n}]$ for any $0 < a < b < 1$, $0 < c < d$, where

$$g_1(x, y) = \frac{x}{\sqrt{2\pi y^3(1-y)}} \exp\left(-\frac{x^2(1-y)}{2y}\right).$$

Theorem 2. Let X_i be strongly arithmetic random variables with $\mathbf{E}X_i = 0$, $\mathbf{D}X_i < \infty$. Then

$$\mathbf{P}(\tau_M \le xn \,|\, M_n = k_n) \to \int_0^x g_1(s, t)\, dt, \; x \in [0, 1], \; n \to \infty,$$

uniformly by $k_n \in \mathbb{Z}$, $k_n/(\sigma\sqrt{n}) \to s > 0$.

Theorem 1 is proved by the use of Sparre–Andersen identity and results of Caravenna (see [2]). Theorem 2 is a corollary of Theorem 1.

2) Large devations.

Let X be i.i.d. r.v. with $\mathbf{E}X_i = 0$ and suppose that $R(h) = \mathbf{E}e^{hX} < \infty$ for $h \in [0, h^+)$. A random variable $X^{(h)}$ is called *conjugate* to X with parameter h if

$$\mathbf{P}(X^{(h)} \le x) = R(h)^{-1} \int_{-\infty}^x e^{ht} \mathbf{P}(X \in dt).$$

Denote by $S_n^{(h)}$ the random walk with i.i.d. steps $X_i^{(h)}$.

It's easy to see that $m(h) = \mathbf{E}X^{(h)}$ exists for any $h \in [0, h^+)$. Moreover, $m(h)$ is strictly increasing on $[0, h^+)$. Let $m^+ = \lim_{h \to h^+} m(h)$.

Theorem 3. Let X_i be strongly arithmetic random variables with $\mathbf{E}X_i = 0$, satifying $\mathbf{E}e^{hX_1} < \infty$, $h \in [0, h^+)$. Then

$$\mathbf{P}(\tau_M = n - l \,|\, M_n = k) \sim g_2\left(\frac{k}{n}, l\right), \; n \to \infty,$$

uniformly by $l \in \mathbb{Z}$, $l \le a$, $k/n \in [c, d]$ for any $a > 0$, $0 < c < d < m^+$, where

$$g_2(x, l) = \frac{\mathbf{P}\left(S_i^{(h_x)} > 0, i \le l\right) R(h_x)^{-l}}{\sum_{j=0}^{\infty} \mathbf{P}\left(S_i^{(h_x)} > 0, i \le j\right) R(h_x)^{-j}}.$$

Theorem 3 is proved by the use of local version of large deviation theorem for M_n, similar to results of Shklyaev (see [3]) and Kozlov (see [4]).

3) Moderate deviations.

Theorem 4. Let X_i be strongly arithmetic random variables with $\mathbf{E}X_i = 0$, satifying $\mathbf{E}e^{hX_1} < \infty$ for some $h > 0$. Then

$$\mathbf{P}(\tau_M = n - l \,|\, M_n = k) \sim \frac{k^2}{\sigma^2 n^2} g_3\left(\frac{lk^2}{n^2\sigma^2}\right), \; n \to \infty,$$

uniformly by $n^{1/2+\delta} < k < n^{1-\delta}$, $lk^2/n^2 \in [\delta, M]$ for any $\delta, M > 0$, where

$$g_3(x) = \frac{1}{\sqrt{2\pi x}} e^{-x/2}$$

is a probability density function of χ^2_1 distribution.

Theorem 5. Suppose X_i satisfy the assumptions of Theorem 4. Then for any $\alpha \in (0.5, 1)$

$$\mathbf{P}\left(\frac{n - \tau_M}{n^{2-2\alpha}} \le x \,\middle|\, M_n = k_n\right) \to \int_0^{x/s} g_3(t)dt, \; n \to \infty,$$

uniformly by $k_n/(\sigma n^\alpha) \to s > 0$.

Theorem 4 is based on local large deviation theorem for random walk (see [5], Chapter 9) and Sparre-Andersen identity. Theorem 5 is a corrolary of Theorem 4.

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Size distributions of regolith granules at meteorite impact on the Moon and in laboratory experiments with microwave discharge

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One of the many unusual phenomena experimentally discovered during landings of spacecraft on the Moon is the existence of dusty plasma clouds, which fill a layer with a thickness of several tens of kilometers above Moon surface. These clouds consist of charged dust grains of regolith, which covers the entire lunar surface with a layer whose thickness reaches several meters in lunar seas [1, 2]. Samples of lunar regolith were delivered to Earth by spacecraft, and its structure was well studied. Regolith is a mixture of powders of different oxides (aluminum oxide, silicon oxide, iron oxide, etc) with a mean grain radius of $70 - 100$ μm, and a large number of particles with the radius of the order of one millimeter. The regolith particle radius distribution is a power-series [3]. The dusty plasma cloud density above the lunar surface and its altitude distribution were not specifically studied during the XX century Moon exploration programmes. However, observations showed that under lunar conditions, charged particles of regolith have increased adhesive properties that limit the use of most spacecraft systems on lunar surface [4]. This is why production of charged dust grain flows in laboratory conditions is intensively studied in order to test the components of future lunar technology [5].

In these experiments, the parameters of grain distributions over altitude, size and velocity can only be obtained from different (plasma) models. However, the modern models, which take into account different physical processes, such as, e.g., the influence of the solar wind,

the photoionization, the shock waves, can only describe the ascension of micron-sized particles to altitudes of about one meter [6].

It is known that the Moon is constantly bombarded by meteorites. The impact of even a small-size meteorite produces enough energy to melt, vaporize and destroy the regolith and release metal and oxygen oxides [7].

Earlier, development of chain plasmachemical reactions producing a dusty plasma cloud above the surface of metal and dielectric powder mixtures irradiated by powerful gyrotron radiation was experimentally observed in the Plasma Physics Department of the Prokhorov General Physics Institute of the Russian Academy of Sciences [8]. The absorbed microwave power necessary to ignite these chemical processes was found to be comparable with the impact of a meteorite with the size of about ten microns. This allowed us to conduct a model laboratory experiment with the goal to create ensembles of dusty structures during the development of chain exothermic plasmachemical reactions initiated by gyrotron in metal-dielectric powder mixtures whose composition imitated lunar regolith.

In these experiments, we used a mixture of oxide powders with a percentage composition the same as in regolith, and with a uniform particle size distribution. Crossing the energy threshold of chain reactions (gyrotron pulse energy of $1-3$ kJ at pulse duration of $1.5-4$ ms) initiates an explosive process caused by Coulomb repulsion of charged particles from regolith surface into the plasmachemical reactor volume. After the powder mixture explosion, self-propagating chain reactions of high-temperature plasmachemical synthesis were observed, which continue for tens of seconds. During this period, the suspended dust grains levitate several tens of centimeters above the powder mixture surface, and produce a levitating cloud, which occupies not only the entire reactor volume, but also raises to a height of up to 1 m above the rector (and this, in the terrestrial gravitation field). The energy produced during this process exceeds the initiation energy several hundredfold. Melted regolith spheroids with diameters of $1-1000\,\mu$m are deposited on the side surface of the reactor above the powder mixture, whose size distribution is also uniform.

In this work, we note the possible analogy between this process and the raise of dusty plasma clouds above regolith surface on the Moon, which, similarly, could be caused by not only physical processes, but also chemical chain processes caused by meteorite impact.

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Simulation of finite-source retrial queueing systems with collisions and blocking

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This paper investigates a retrial queuing system with a single server, which is liable to random breakdowns (Bérczes and Sztrik and Tóth and Nazarov [1], Kim [2], Wang and Zhao and Zhang [5]) by the help of a simulation program. The number of sources of calls is finite and collision (Nazarov and Kvach and Yampolsky [3], Tóth and Bérczes and Sztrik and Kvach [4]) can take place. The failure of server block the system's operation therefore the arriving customers can not enter the system, meaning that those calls are lost. All the random variables included in the model construction are assumed to be generally distributed and independent of each other. The novelty of this analysis is the inspection of blocking effect on the performance measures using different distributions. Various figures represent the impact of different distributions on the main performance measures like mean and variance of number of customers in the system, mean and variance of response time, mean and variance of time a customer spent in service, mean and variance of sojourn time in the orbit.

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Permutation tests for homogeneity based on some characterizations

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One of the important problems in statistics is the problem of testing the equality of the distributions of several populations. A typical example, often referred to, is the comparison of several drugs with a placebo, where the hypothesis of no drug effect is tested versus the alternative of at least one effect. There is a number of tests for this problem especially for the two-sample problem. Such procedures are usually not distribution free, the distribution of the test statistic depends on the distributions of the samples, therefore critical points for the distribution of

the test statistic under the null hypothesis of homogeneity are obtained using a bootstrap-type resampling scheme, see, for example, Allen [1].

A class of tests for testing the homogeneity of two populations is proposed by Meintanis [2]. The tests are based on the empirical characteristic function, and the test procedure is based on resampling from the permutation distribution of the test statistic. The test statistic is the weighted L^2 distance between empirical characteristic functions. Weight functions of two types are used – a normal density and a Laplace density.

In this work, we propose tests of homogeneity of two or more distributions. The tests are based on characterizations of homogeneity obtained by Ushakov [3] and Ushakov and Ushakov [4]. Since the distribution of the test statistics depends on the distributions of populations, we also use the bootstrap-type resampling technique, mentioned above.

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On stability of characterization of distribution types

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The following problem often arises in applications. Suppose that there are a number of small independent samples such that in each small sample observations are independent and identically distributed while from sample to sample they have different values of location parameter. First this problem was posed by A.N. Kolmogorov, see Zinger [1]. In

this situation it is necessary to use statistics which do not depent on the location parameter. Reconstruction of the type of initial distribution from distribution of such a statistic is an actual problem, in particular for goodness of fit testing.

The stability of the reconstruction has been studied by a number of authors, see, for example, Prokhorov [2], Zinger and Kagan [3], Kagan and Klebanov [4]. Ushakova [5] proved that the upper bound of stability has the order $\epsilon^{1/3}L(\epsilon)$, where $L(\epsilon)$ is a slowly varying function. Here this estimate is improved.

In what follows we suppose (without loss of generality) that the small subsamples have size 3, i.e. the minimal necessary size. The main result is as follows. Let X_1, X_2, X_3 be independent random variables with common distribution function $F(x-\theta)$ and unit variance, $\Phi(x)$ be the standard normal distribution function. Let $a = (a_1, a_2, a_3)$ be a vector satisfying the following conditions:

$$a_1 + a_2 + a_3 = 0, \quad a_1^2 + a_2^2 + a_3^2 = 1.$$

Denote

$$X^{(a)} = a_1 X_1 + a_2 X_2 + a_3 X_3,$$

and let $F^{(a)}(x)$ be the distribution function of $X^{(a)}$.

Theorem. *If*

$$\sup_a \int_{-\infty}^{\infty} (1+|x|) d|F^{(a)} - \Phi| \le \epsilon < 1,$$

then

$$\sup_x |F(x-\theta) - \Phi(x)| \le \epsilon^{1/2} L(\epsilon)$$

for some θ, where $L(\epsilon)$ is a slowly varying function.

We also consider the problem of testing for homogeneity of two samples in the considered case when the samples consist of small subsamples.

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Development of a semiautomatic system for processing the magnetic probe diagnostic data on L-2M stellarator

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At the present time, experiments on plasma heating and confinement are carried out on the L-2M stellarator (Prokhorov General Physics Institute of the Russian Academy of Sciences) using MIG-3 gyrotron complex which allows us to achieve record specific energy deposition into the plasma (up to 3 MW/m3) [1]. The development of this heating complex as well as studies of the new pulsed-periodic regime of plasma heating, during which the 10-ms-long gyrotron pulse is separated into train of three 3-ms-long pulses [2] increases the necessity of studying the stability of the signals of both macro (temperature, density, energy deposition) and micro (signal fluctuations, turbulence diagnostics) parameters during the analysis of experimental data [3, 4]. Among the latter is the magnetic probe diagnostic of the L-2M stellarator, which consists of a series of up to 9 detectors placed in different diagnostic cross-sections and allows us to study the spectral characteristics of different spatial modes of the low-frequency plasma turbulence.

In this work, analysis of the signals of magnetic probe diagnostics of the L-2M stellarator in the above regimes is presented, using the semi automatic processing system that is being developed [5].

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Multivariate regular variation in probability theory

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It is well-known that the regularly varying functions of one variable were introduced by J. Karamata (1930). Namely, positive measurable function $f(t)$, defined for $t \geq C \geq 0$ is said to be regularly varying at infinity iff, for any $\lambda > 0$, there exists a positive and finite

$$\lim_{t\to\infty} \frac{f(\lambda t)}{f(t)} = \varphi(\lambda) \quad (\Rightarrow \varphi(\lambda) = \lambda^{\varrho}).$$

A number ϱ is called as the index of regular variation of the function $f(t)$.

If in the one-dimensional case, there is in fact one definition of regularly varying functions, then in the multidimensional case the situation is significantly different. Namely, there are a number of classes of functions that generalize this concept. At the same time, each class of such functions has (generally speaking), its own applications.

Multidimensional extensions of regularly varying functions are defined by different authors, for instance: Bajšanski and Karamata (1969), Yu. Drozhzhinov and B. Zav'yalov (1984, 1986, 1994), L. de Haan (1985), S.M. Kozlov (1983), A. Nagaev and A. Zaigraev (2003), M. Meershaert (1986, 2001), I.S. Molchanov (1993), E. Omey (1982, 1989), T. Ostrogorsky (1995, 1997, 1998), A. Stam (1977), S. Resnick (1986, 2007, 2015), E. Rvaĉeva (1962), A. Yakymiv (1981, 2003, 2018).

In addition, a number of articles by different authors containe concrete results in such areas of probability theory and it's applications as random point processes, extreme values, the summation theory of random vectors, generalized renewal theory, branching processes, infinitely divisible random vectors, finance mathematics, risk theory, random permutations and random mappings, random vectors with multiple power series distribution, growth of preferential attachment networks and others. Some references one can see in the author's book (2005).

It is less known that J. Karamata, together with B. Bajšanski (1969) gave a deep generalization of regularly varying functions not only to the multidimensional case, but also for topological groups. Namely, continuous functions $f\colon G \to R_+$ are considered, where G is an arbitrary topological group where a filter $\mathfrak{U}$ of open convex sets in G with countable base is given. The filter $\mathfrak{U}$ is thought of as G-invariant, that is, $Uh \in \mathfrak{U}$ and $hU \in \mathfrak{U}$ for any set $U \in \mathfrak{U}$ and any element $h \in U$. A function f is said to be regularly varying with respect to filter $\mathfrak{U}$ if the limit

$$\lim_{g\to\infty} \frac{f(gh)}{f(g)} = \phi(h)$$

exists for any $h \in G$, where $g \to \infty$ means convergence with respect to the filter. In this paper, a theorem about uniform convergence is also proved.

In Ostrogorsky (1995, 1997, 1998), the research started in Bajšanski, Karamata (1969) is continued. As the group G, various cones in $\mathbf{R}^n$ are considrered, such as the hyper-octant, the future light cone, arbitrary homogeneous cones.

In Drozhzhinov and Zav'yalov (1984) and further papers the regularly varying generalized functions with support on homogeneous cones

were introduced and studied, see also Vladimirov, Drozhzhinov and Zav'yalov's book (1986).

In the Omey's doctor's dissertation (1982) and his book (1989), measurable functions : $\mathbf{R}_+^2 \to R_+$ are studied such that the limit

$$\lim_{t\to\infty} \frac{f(r(t)x, s(t)y)}{f(r(t), s(t))} = \lambda(x, y)$$

exists for some auxiliary functions $r, s\colon R_+ \to R_+$, $r(t) \to \infty$, $s(t) \to \infty$ as $t \to \infty$, some positive function $\lambda(x, y)$ and for all $x, y > 0$. Using this definition, E. Omey obtained some results in generalized renewal theory, extreme value theory and for domains of attraction of partial sums of i.i.d. random vectors.

In Meershaert (1986, 1988), functions $f(t)$ of one variable t are considered whose values are non-singular linear operators from $\mathbf{R}^k$, and the idea of regular variation is extended to this case. In Meershaert and Scheffler (2001), the limit distributions for sums of i.i.d. random vectors with operator normalization were obtained with a number of applications, with helping of such notion.

In I.S. Molchanov (1993), regularly varying functions $f(x)$ defined in some m-dimensional cone are introduced whose values are closed (compact) sets in $\mathbf{R}^d$. Further, the limit behaviour of such random sets were investigated.

According to S. Resnick (1986), a random vector X taking values in $\mathbf{R}^n$ is said to be regularly varying at infinity with index $\alpha \geq 0$ and spectral (probability) distribution P_s on the unit sphere $S^{n-1} \subset \mathbf{R}^n$ if there exist positive c and σ_k, $k \in N$, such that, as $k \to \infty$,

$$k\,\mathsf{P}\{\sigma_k^{-1} X \in A(r, B)\} \to cr^{-\alpha} P_s(B)$$

for all sets $B \subset S^{n-1}$ of continuity of the limiting measure P_s and $r > 0$, where

$$A(r, B) = \{x\colon x \in \mathbf{R}^n,\ |x| > r,\ x/|x| \in B\}.$$

In Basrak, Davis, Mikosh (2002), it is shown that if a random vector X regularly varies at infinity with index $\alpha > 0$, then for any $x \in \mathbf{R}^n$ and some slowly varying at infinity function $L(t)$ there exists the limit

$$\lim_{t\to\infty} \frac{\mathsf{P}\{(x, X) > t\}}{t^{-\alpha} L(t)} = \omega(x),$$

and there exists $x_0 \neq 0$ such that $\omega(x_0) > 0$. It is also shown that for non-integer $\alpha > 0$ the corresponding converse assertion is true, while

the limiting measure P_s is uniquely determined by the function $\omega(x)$. A counterexample is given for $\alpha = 2$.

In A. Nagaev and A.Źaigraev (2003), a function $f(x)$, $x \in \mathbf{R}^n$, is said to be (β, λ) regularly varying if, as $|x| \to \infty$,

$$\sup_{e_x \in E_\lambda} \left| \frac{f(x)}{r_\beta(|x|)} - \lambda(e_x) \right| = o(1),$$

where $e_x = x/|x|$, $r_\beta(t)$ regularly varies as $t \to \infty$ with

$$E_\lambda = \{a \in S^{n-1} \colon \lambda(a) > 0\}.$$

In Resnick (2007, 2008), some classes of measures and functions are considered. In particular, they allow to obtain the next asymptotics:

$$t\,\mathsf{P}\{(X/a(t) \geq x, Y/b(t) \geq y)\} \to \mu_0(x,y) \equiv$$

$$\equiv \mu([x,\infty] \times [y,\infty]), \quad \forall x, y > 0,$$

where r.v. X and Y have applications and concrete interpretation in preferential attachment networks, see Resnick et al (2015, 2016). The authors of last two papers say that that the regular variation is nonstandard, if $a(t)$ and $b(t)$ have different order at infinity.

Let $U = \{U_k,\ k \in I \subseteq [0,\infty)\}$ stand for an arbitrary family of linear operators in $\mathbf{R}^n$ which leave invariant the cone $\Gamma \subseteq \mathbf{R}^n$:

$$U_k \Gamma = \Gamma \quad \forall k \in I.$$

We assume that ∞ is a limit point of the set I. According to Yakymiv (2003), a function $f(x)$, which is defined, positive, and measurable in Γ, is regularly varying in Γ along a family $U = \{U_k,\ k \in I\}$ iff for some vector $e \in \Gamma$ and all $x \in \Gamma$ as $x_k \to x$, $k \to \infty$, $k \in I$,

$$\frac{f(U_k x_k)}{f(U_k e)} \to \phi(x) > 0, \qquad \phi(x) < \infty.$$

In the already mentioned paper, this definition was used at the study of asymptotics of infinitely divisible distributions with a support in homogeneous cones. Also the asymptotic properties of some classes of random permutations and random mappings were investigated (2009, 2010, 2014). Recently (2018), we obtained the limit theorems (integral and local) for multiple power series distributions. (In the last two applications, this definition is used in the case when $\Gamma = \mathbf{R}^n_+$ and operators U_k are diagonal).

In this abstract, we mention only different multidimensional generalizations of the regularly varying functions known to the author and some their applications in probability theory. But we suppose to give much more information in this direction at the presentation.

Magnetoencephalography inverse ill-posed problem

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This paper continues a series of studies dealing with noninvasive preoperative methods for localizing eloquent areas of the human brain. Magnetoencephalography (MEG) is a noninvasive method for studying brain activity. It has high temporal and spatial resolutions, and only weakly depends on the inhomogeneities of the head conductivity, which makes it a valuable tool for both neuroscience and clinical applications [1].

The inverse problem of magnetoencephalography is ill-posed and difficult for both analytical and numerical solutions. Additional complications arise from the volume (passive) currents and the associated magnetic fields, which strongly depend on the brain geometry.

An analytical formula is derived for the solution of the forward problem that computes the magnetic field on the surface of the head from the known location and orientation of a current dipole in the low-frequency approximation in the spherical model [2].

In this paper we find approximate analytical solutions for the forward and the inverse problems in the spheroid geometry. We compare the obtained results with the exact solution of the forward problem and deduce that for a wide range of parameters our approximation is valid.

In addition, the paper considers the question of the stability of solutions of the inverse problem of MEG to the effect of noise. The solution is unstable to the effect of noise on its angular component, but the deviation from the true solution is much less than the noise variance.

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The application of the ICA method and window dispersion in the study of bioequivalence of drugs

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The original drug is a drug that differs from all previously registered drugs with a pharmacologically active substance (pharmaceutical substance) or a combination of such substances.

Generic medicinal product (generic drug) is a drug that contains the same pharmacologically active substance (pharmaceutical substance) in the same dose and the same dosage form as the original drug, is equivalent to the original product in terms of quality, efficiency and profile security and is produced without a license of the company owning the original medicinal product. Implemented after the expiry of the patent or other exclusive rights to manufacture and sale of the original.

Thus, the presence on the market of generic funds is due, first of all, the expiry of the terms of patent protection for the production of original funds. The term of patent protection of a medicinal product, as a rule, is not more than 20 years.

Generic medicines must meet the same quality, efficacy and safety standards as the original medicines, but in addition, convincing evidence

must be provided that they are equivalent, previously registered similar medicines and are clinically interchangeable with them.

To date, the main criterion for evaluating bioequivalence is the level of drug concentration in the blood over time. To do this, experiments are performed on healthy volunteers, after which the results are averaged and the concentration-time curve is plotted against time. The conclusion about bioequivalence of drugs is made on the basis of the results obtained by comparing the area under the resulting curves.

However, this method has a huge disadvantage: it is impossible to trace and take into account the main stages of the kinetics of the preparation, and even with a sufficiently large coincidence of areas under the curves, it can not be concluded that the original drug and generic behave in the human body in the same way [3].

In this paper, the method of estimating bioequivalence, whose main goal is to break the drug concentration curve in the body into components, is considered, implying that this curve is a signal that demonstrates the behavior of the drug. These components are directly related to the main stages of the drug. Denoting the boundaries of these stages, we can, with a minimum of error, compare drugs by the duration and nature of these stages. To isolate the components, methods such as the method of independent components, the window dispersion method[1], and the study of the variance gamma process will be used.

The window dispersion is defined by the formula [2]:

$$W_{N,t} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - X)^2,$$

where

$$X = \frac{1}{N}\sum_{i=1}^{N} X_i.$$

The window size was chosen empirically, the best results were obtained with a window width equal to 3 points of reference. Window dispersion removes the trend and increases the differences in the concentration values, so a sharp decrease in the window dispersion will correspond to a decrease in the concentration, which is the case with absorption.

The method of independent components (OLS, Independent component analysis, ICA) [4] is a method for dividing a multidimensional signal into additive components. Suppose we have signal sources s_1, s_2, ..., s_n and signal receivers x_1, x_2, ..., x_n. Each receiver captures the weighted

sum of the signals.

$$x_1(t) = a_{11}\, s_1(t) + a_{12}\, s_2(t) + ... + a_{1n}\, s_n(t),$$
$$x_2(t) = a_{21}\, s_1(t) + a_{22}\, s_2(t) + ... + a_{2n}\, s_n(t),$$
$$... \quad ... \quad ... \quad ...$$
$$x_n(t) = a_{n1}\, s_1(t) + a_{n2}\, s_2(t) + ... + a_{nn}\, s_n(t),$$

where t is a fixed instant of time.

Our task is to determine the values of the sources from the values of the receivers s, $i = 1, ..., n$ and weights a_{ij}, $i = 1, ..., n$, $j = 1, ..., n$. Sources of signal are also called hidden variables, latent variables or independent components.

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On multivariate models based on scale mixtures

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Let a multivariate random variable $X = (X_1, ..., X_n)$ have independent components X_j. Further assume that every component X_j has the d-dimensional strictly stable, or geometrically strictly stable distribution with the stability index α_j $(1 < \alpha_j \leq 2)$ and the characteristic function

$$\phi_j(\theta_j) = E\left(\exp(i\,\theta_j^T X_j)\right), \quad \theta_j = (\theta_j^{(1)}, ..., \theta_j^{(d)}) \in \mathbb{R}^d.$$

Thereby X is both multivariate and multidimensional random variable.

Consider the vector

$$Y = (y_1, ..., y_n) = (\beta_1^{1/\alpha_1} X_1, ..., \beta_1^{1/\alpha_n} X_n),$$

which is a scale mixture of the above random variable X and the vector $\beta^* = (\beta_1^{1/\alpha_1}, ..., \beta_n^{1/\alpha_n})$, or, in other words, β^* is the subordinator of X.

The positive random vector β mentioned above is given by its Laplace transform

$$\Phi(s_1, ..., s_n) = E\left(\exp(-\sum_{j=1}^{n} s_j \beta_j)\right).$$

Theorem.

The characteristic function of the random variable Y is

$$\psi_Y(\theta_1, ..., \theta_n) = \Phi(-\log \phi_1(\theta_1), ..., -\log \phi_n(\theta_n)).$$

As examples of using the explicit expression of the characteristic function the following subordinators were considered:

– Marshall–Olkin multivariate exponential distribution.

– Multivariate gamma distribution.

In the first case, the distribution is a discrete mixture of the generalized multivariate Linnik distribution [1] and its mixtures with the distributions of its own projections onto all the coordinate hyperplanes.

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